

3.6 Start Thinking

Graph the lines $y = x$ and $y = 5x$. Note the change in slope of the line. Graph the line $y = 20x$. What is happening to the line?

What would the line look like if the slope was changed to 100? 1000? What if the slope was the greatest number you can think of? Explain how this shows the slope of a vertical line is *undefined*.

3.6 Warm Up

Graph the point and its image in a coordinate plane.

1. $P(-5, 3)$; reflected in the y -axis
2. $Q(-4, -3)$; reflected in the x -axis
3. $R(-1, -5)$; reflected in the line through $(-2, 2)$ and $(-2, 3)$
4. $S(5, 1)$; reflected in the line through $(3, -3)$ and $(8, -3)$

3.6 Cumulative Review Warm Up

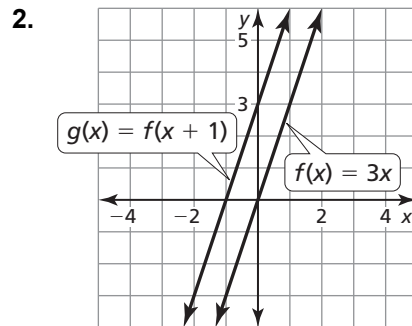
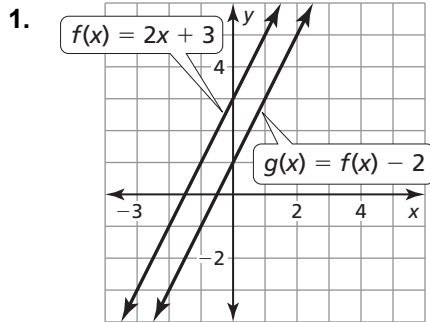
Solve the equation.

1. $4c - 2 = 2c$
2. $3(7q + 6) = 3q$
3. $5(-g - 10) = 6 - 13g$
4. $m + 4 = 12$
5. $5 = 6w - 9w - 1$
6. $3k + 2(3k + 2) = 49$

3.6

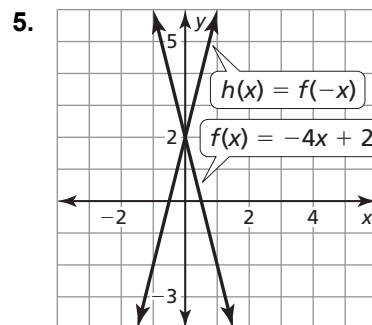
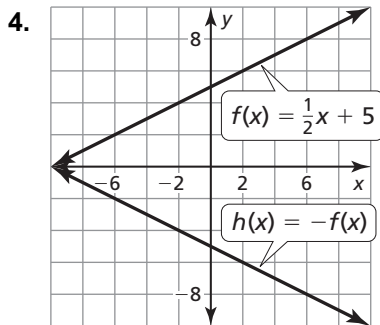
Practice A

In Exercises 1 and 2, use the graphs of f and g to describe the transformation from the graph of f to the graph of g .



3. You and a friend start running from the same location. Your distance d (in miles) after t minutes is $d(t) = \frac{1}{7}t$. Your friend starts running 10 minutes after you. Your friend's distance f is given by the function $f(t) = d(t - 10)$. Describe the transformation from the graph of d to the graph of f .

In Exercises 4 and 5, use the graphs of f and h to describe the transformation from the graph of f to the graph of h .



In Exercises 6 and 7, use the graphs of f and r to describe the transformation from the graph of f to the graph of r .

6. $f(x) = x + 2$; $r(x) = f(3x)$ 7. $f(x) = 3x + 6$; $r(x) = \frac{1}{3}f(x)$

In Exercises 8 and 9, write a function g in terms of f so that the statement is true.

8. The graph of g is a vertical translation 3 units down of the graph of f .
9. The graph of g is a reflection in the x -axis of the graph of f .

3.6 Practice B

In Exercises 1 and 2, use the graphs of f and g to describe the transformation from the graph of f to the graph of g .

1. $f(x) = -x - 3; g(x) = f(x + 5)$ 2. $f(x) = \frac{1}{3}x - 2; g(x) = f(x - 6)$

3. The total cost C (in dollars) to rent a 14-foot by 20-foot canopy for d days is given by the function $C(d) = 15d + 30$, where the setup fee is \$30 and the charge per day is \$15. The setup fee increases by \$20. The new total cost T is given by the function $T(d) = C(d) + 20$. Describe the transformation from the graph of C to the graph of T .

In Exercises 4 and 5, use the graphs of f and h to describe the transformation from the graph of f to the graph of h .

4. $f(x) = -3 - x; h(x) = f(-x)$ 5. $f(x) = \frac{1}{3}x + 1; h(x) = -f(x)$

In Exercises 6 and 7, use the graphs of f and r to describe the transformation from the graph of f to the graph of r .

6. $f(x) = 5x - 10; r(x) = f\left(\frac{2}{5}x\right)$ 7. $f(x) = -\frac{1}{3}x + 2; r(x) = 6f(x)$

In Exercises 8–11, use the graphs of f and g to describe the transformation from the graph of f to the graph of g .

8. $f(x) = -3x + 5; g(x) = f(x - 3)$ 9. $f(x) = -2x + 6; g(x) = f\left(\frac{4}{3}x\right)$

10. $f(x) = 4x - 3; g(x) = \frac{1}{2}f(x)$ 11. $f(x) = -2x; g(x) = f(x) + 3$

In Exercises 12 and 13, write a function g in terms of f so that the statement is true.

12. The graph of g is a horizontal shrink by a factor of $\frac{2}{3}$ of the graph of f .

13. The graph of g is a horizontal translation 5 units left of the graph of f .

In Exercises 14–17, graph f and h . Describe the transformations from the graph of f to the graph of h .

14. $f(x) = x; h(x) = -2x + 1$ 15. $f(x) = x; h(x) = \frac{3}{2}x + 2$

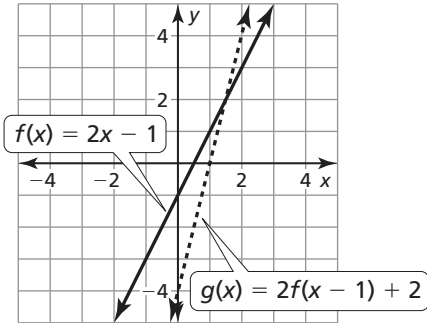
16. $f(x) = 2x; h(x) = 8x - 3$ 17. $f(x) = 3x; h(x) = -3x - 5$

3.6

Enrichment and Extension

Multiple Transformations of Linear Equations

Example: Let $f(x) = 2x - 1$. Graph the transformation $g(x) = 2f(x - 1) + 2$. Use composition of functions to rewrite $g(x)$. Then find $g(-1)$, $g(0)$, and $g(1)$ to check your graph.



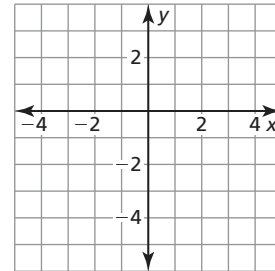
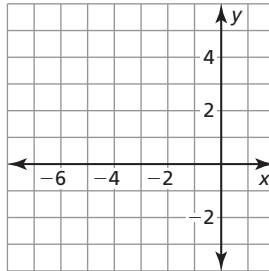
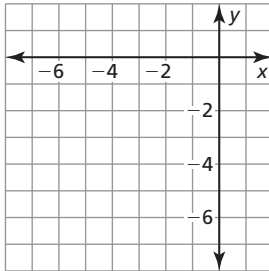
$$g(x) = 2(2(x - 1) - 1) + 2 \qquad g(-1) = -8$$

$$g(x) = 2(2x - 3) + 2 \qquad g(0) = -4$$

$$g(x) = 4x - 4 \qquad g(1) = 0$$

Let $f(x) = 3x + 2$. Graph each transformation given. Use composition of functions to rewrite $g(x)$. Then find $g(-2)$, $g(0)$, and $g(1)$.

1. $g(x) = -f\left(\frac{1}{3}x\right) - 4$
2. $g(x) = \frac{1}{5}f(x + 3) + 2$
3. $g(x) = f(2x - 1) - 3$



4. What is different about Exercise 3? Is it possible to write a rule for this type of transformation? If so, please demonstrate.



Puzzle Time

What Did One Watch Say To The Other Watch?

Write the letter of each answer in the box containing the exercise number.

Describe the transformations from the graph of f to the graph of g .

1. $f(x) = x - 7$; $g(x) = f(x - 2)$
2. $f(x) = \frac{3}{5}x + 9$; $g(x) = f(-x)$
3. $f(x) = -6x - 11$; $g(x) = f\left(\frac{1}{4}x\right)$
4. $f(x) = \frac{2}{3}x - 18$; $g(x) = f(x) - 2$
5. $f(x) = -10x + 21$; $g(x) = 8f(x)$
6. $f(x) = x$; $g(x) = x + 6$
7. $f(x) = x$; $g(x) = -x + \frac{9}{16}$
8. $f(x) = x$; $g(x) = \frac{1}{2}x - 6$
9. $f(x) = x$; $g(x) = 4x - 3$
10. Members of the marching band need to rent a moving van to haul their instruments back and forth to several competitions. The total cost C (in dollars) to rent a moving van for m miles is given by the function $C(m) = 4m + 225$, where the flat fee is \$225 and the charge per mile is \$4. The flat fee decreases by \$5. The new total cost T is given by the function $T(m) = C(m) - 5$. Describe the transformation from the graph of C to the graph of T .

Answers

- G. vertical stretch by a factor of 8
- I. reflection in the x -axis and a vertical translation $\frac{9}{16}$ units up
- A. reflection in the y -axis
- T. horizontal stretch by a factor of 2 and a vertical translation 6 units down
- N. vertical translation 5 units down
- U. vertical translation 2 units down
- E. horizontal translation 2 units right
- M. horizontal shrink by a factor of $\frac{1}{4}$ and a vertical translation 3 units down
- T. vertical translation 6 units up
- O. horizontal stretch by a factor of 4

5	3	8		2		9	7	10	4	6	1
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