6.6 Start Thinking

Find the result of $256 \cdot \frac{1}{4}$. Continue multiplying each result by $\frac{1}{4}$ until you reach a result of 1. Write each result in a list.

Is the list getting larger or smaller? Why? If the list continues, what will the numbers look like? Is it possible to get negative numbers in the list? Why or why not?

6.6 Warm Up

Find the common difference in the sequence.

1. -2, 0, 2, 4, ...**2.** -9, -8, -7, -6, ...**3.** 0.03, 0.09, 0.15, 0.21, ...**4.** 12, 10, 8, 6, ...**5.** 2, $3\frac{1}{4}$, $4\frac{1}{2}$, $5\frac{3}{4}$, ...**6.** -4, -7, -10, -13, ...

6.6 Cumulative Review Warm Up

Determine whether the graph represents a *linear* or *nonlinear* function. Explain.





6.6 Practice A

In Exercises 1–3, find the common ratio of the geometric sequence.

1. 2, 6, 18, 54, ... **2.** 135, 45, 15, 5, ... **3.** 7, -14, 28, -56, ...

In Exercises 4–6, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

4. 1, 4, 9, 16, ... **5.** 12, 17, 22, 27, ... **6.** 4, -12, 36, -108, ...

In Exercises 7 and 8, determine whether the graph represents an *arithmetic* sequence, a geometric sequence, or *neither*. Explain your reasoning.



In Exercises 9 and 10, write the next three terms of the geometric sequence. Then graph the sequence.

9. 3, 15, 75, 375, ... **10.** 1024, -256, 64, -16, ...

In Exercises 11–14, write an equation for the *n*th term of the geometric sequence. Then find a_6 .

11. 3, 6, 12, 24, ...

12. 0.375, 3, 24, 192, ...

 n
 1
 2
 3
 4

 a_n 0.0124
 1.24
 124
 12,400

14.	n	1	2	3	4
	an	-1024	128	-16	2

- **15.** A digital city map displays an area of 544 square units. After you zoom in once, the area is 272 square units. After you zoom in a second time, the area is 136 square units. What is the area after you zoom in five times?
- **16.** What is the 8th term of the geometric sequence where $a_2 = 20$ and r = 5?

6.6

Practice B

In Exercises 1–3, find the common ratio of the geometric sequence.

1. 5, 20, 80, 320, ... **2.** 144, -72, 36, -18, ... **3.** 24, 84, 294, 1029, ...

In Exercises 4–7, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

4. 2.786, 27.86, 278.6, 2786, ...**5.** 86, 71, 56, 41, ...**6.** 4, -10, 16, -28, ...**7.** 112, -28, 7, $-\frac{7}{4}$, ...

In Exercises 8 and 9, write the next three terms of the geometric sequence. Then graph the sequence.

8. -2, -12, -72, -432, ... **9.** $\frac{54}{25}$, $\frac{18}{5}$, 6, 10, ...

In Exercises 10–13, write an equation for the *n*th term of the geometric sequence. Then find a_6 .

10. $\frac{3}{125}$, $\frac{3}{25}$, $\frac{3}{5}$, 3, ... **11.** 0.2, 1.6, 12.8, 102.4, ...

12.	n	1	2	3	4	13.	n	1	2	3	4
	a _n	2436	-243.6	24.36	-2.436		a _n	-1458	-162	-18	-2

14. An archery competition begins with 256 competitors. After the first round, one-fourth of the competing group remains. After the second round, one-fourth of the now smaller competing group remains. The last round is when there are fewer than five members in the competing group.

- **a.** Which round is the last round?
- **b.** How many competitors are in the last round?
- **15.** What is the 10th term of the geometric sequence where $a_3 = \frac{8}{3}$ and $r = \frac{2}{3}$?
- **16.** Find the sum of the terms of the geometric sequence

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$$

Explain your reasoning.

6.6 Enrichment and Extension

Geometric Series

For a geometric sequence with first term a_1 and common ratio r, the sum of the first n terms is given by $S_n = \frac{a_1(1-r^n)}{1-r}$. The sum S of an infinite geometric series with |r| < 1 is given by $S = \frac{a_1}{1-r}$. If $|r| \ge 1$, the series will have no sum.

Example: Find the sum of the first four terms of the geometric series $-3 + 1 - \frac{1}{3} + \frac{1}{9} + \cdots$.

$$a_1 = -3, n = 4, r = -\frac{1}{3};$$
 So, $S_4 = \frac{-3\left(1 - \left(-\frac{1}{3}\right)^4\right)}{1 - \left(-\frac{1}{3}\right)} = -\frac{20}{9}$

In Exercises 1–3, find the sum of the first *n* terms of the geometric sequence.

- **1.** $a_1 = 3, n = 5, r = -2$
- **2.** $a_1 = 8, n = 4, r = \frac{2}{5}$

3.
$$a_1 = 5, a_4 = \frac{5}{8}, n = 10$$

In Exercises 4–6, find the sum of the infinite geometric series, if it exists.

- **4.** $3 \frac{3}{2} + \frac{3}{4} \frac{3}{8} + \cdots$
- **5.** $-1 4 16 32 \cdots$

6.
$$x + \frac{x}{2} + \frac{x}{4} + \cdots$$

7. Find the first term of the series if $S_5 = 27$ and r = -3.



How Does A Penguin Make Pancakes?

Write the letter of each answer in the box containing the exercise number.

Find the common ratio of the geometric sequence.

1.	5, 20, 80, 320,	2.	49, 7, 1, $\frac{1}{7}$,							
3.	$\frac{2}{9}$, -2, 18, -162,	4.	0.023, 0.23, 2.3, 23,							
5.	$\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \dots$	6.	6, 18, 54, 162,							
Write	the next three terms of th	ne ge	eometric sequence.							
7.	192, 48, 12, 3,	8.	6, 12, 24, 48,							
9.	-100, 10, -1, 0.1,	10.	-500, -100, -20, -4,							
Write Then	an equation for the <i>n</i> th te find a ₆ .	ərm	of the geometric sequence.							
11.	3, 12, 48, 192,	12.	2.88, 1.44, 0.72, 0.36,							
13.	$-\frac{1}{81}, -\frac{1}{27}, -\frac{1}{9}, -\frac{1}{3}, \dots$	14.	-256, 64, -16, 4,							
15.	15. There were 32 chess players in the competition. After the first round, 16 players remained. After the second round, 8 players remained. Write an equation for the <i>n</i> th term of the geometric sequence. How many players remained after the fifth round?									

An	swers
S.	3
I.	$\frac{1}{7}$
н.	2
L.	10
R.	3072
Е.	4
I.	-3
F.	0.09
Т.	$-\frac{1}{2}$
Ρ.	$\frac{1}{4}$
т.	-9
Ρ.	-0.01, 0.001, -0.0001
S.	$\frac{3}{4}, \frac{3}{16}, \frac{3}{64}$
W.	$-\frac{4}{5}, -\frac{4}{25}, -\frac{4}{125}$
I.	96,192,384

10	8	3	15	2	5	7	12	4	13	9	14	1	11	6