

## 6.7 Start Thinking

Use the sequence  $-12, -14, -16, -18, \dots$  to complete the table.

Term	Term number	Common difference
-12		
-14		
-16		
-18		

The recursive rule  $a_1 = -12, a_n = a_{n-1} - 2$  represents the sequence above. Explain this rule as it relates to the sequence. Can you use this rule to determine the term preceding  $-12$ ? If so, what is it?

## 6.7 Warm Up

Find the next three terms in the sequence.

1.  $1, -16, -33, -50, \dots$
2.  $6, 7, 8, 9, \dots$
3.  $-39, -13, -\frac{13}{3}, -\frac{13}{9}, \dots$
4.  $0.5, 1.5, 2.5, 3.5, \dots$
5.  $-1, -8, -64, -512, \dots$
6.  $5, 16, 27, 38, \dots$

## 6.7 Cumulative Review Warm Up

Write a linear function  $f$  with the given values.

1.  $f(3) = -3, f(2) = 0$
2.  $f(-3) = 1, f(7) = -4$
3.  $f(-2) = 0, f(14) = 4$
4.  $f(3) = -2, f(5) = 4$
5.  $f(-2) = 15, f(0) = 9$
6.  $f(1) = 0.3, f(0) = 2.3$

**6.7****Practice A**

In Exercises 1 and 2, determine whether the recursive rule represents an *arithmetic sequence* or *geometric sequence*.

1.  $a_1 = 3; a_n = a_{n-1} + 4$

2.  $a_1 = 3; a_n = 9a_{n-1}$

In Exercises 3–6, write the first six terms of the sequence. Then graph the sequence.

3.  $a_1 = 0; a_n = a_{n-1} + 3$

4.  $a_1 = 18; a_n = a_{n-1} - 8$

5.  $a_1 = 1; a_n = 5a_{n-1}$

6.  $a_1 = 4; a_n = 2.5a_{n-1}$

In Exercises 7 and 8, write a recursive rule for the sequence.

7. 

$n$	1	2	3	4
$a_n$	4	28	196	1372

8. 

$n$	1	2	3	4
$a_n$	6	11	16	21

In Exercises 9 and 10, write an explicit rule for the recursive rule.

9.  $a_1 = -10; a_n = a_{n-1} + 5$

10.  $a_1 = 14; a_n = -2a_{n-1}$

In Exercises 11 and 12, write a recursive rule for the explicit rule.

11.  $a_n = 5(2)^{n-1}$

12.  $a_n = -7n + 3$

In Exercises 13 and 14, graph the first four terms of the sequence with the given description. Write a recursive rule and an explicit rule for the sequence.

13. The first term of the sequence is 8. Each term of the sequence is 12 more than the preceding term.

14. The first term of the sequence is 81. Each term of the sequence is one-third the preceding term.

In Exercises 15 and 16, write a recursive rule for the sequence. Then write the next two terms of the sequence.

15. 3, 5, 8, 13, 21, ...

16. 24, 20, 4, 16, -12, ...

17. Write the first five terms of the sequence  $a_1 = 4; a_n = \frac{1}{2}a_{n-1} + 6$ . Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

## 6.7

## Practice B

In Exercises 1 and 2, determine whether the recursive rule represents an *arithmetic sequence* or *geometric sequence*.

1.  $a_1 = 5; a_n = 12a_{n-1}$

2.  $a_1 = 6; a_n = a_{n-1} - 3$

In Exercises 3–6, write the first six terms of the sequence. Then graph the sequence.

3.  $a_1 = 10; a_n = a_{n-1} - 7$

4.  $a_1 = 36; a_n = -1.5a_{n-1}$

5.  $a_1 = 120; a_n = \frac{1}{5}a_{n-1}$

6.  $a_1 = -6; a_n = -3a_{n-1}$

In Exercises 7 and 8, write a recursive rule for the sequence.

7.

$n$	1	2	3	4
$a_n$	23	13	3	-7

8.

$n$	1	2	3	4
$a_n$	256	128	64	32

In Exercises 9 and 10, write an explicit rule for the recursive rule.

9.  $a_1 = 8; a_n = -9a_{n-1}$

10.  $a_1 = 5; a_n = a_{n-1} + 18$

In Exercises 11 and 12, write a recursive rule for the explicit rule.

11.  $a_n = 1.2n + 2$

12.  $a_n = -76\left(\frac{3}{2}\right)^{n-1}$

In Exercises 13 and 14, graph the first four terms of the sequence with the given description. Write a recursive rule and an explicit rule for the sequence.

13. The first term of the sequence is  $-2$ . Each term of the sequence is  $-5$  times the preceding term.

14. The first term of the sequence is  $23$ . Each term of the sequence is  $9$  less than the preceding term.

In Exercises 15 and 16, write a recursive rule for the sequence. Then write the next two terms of the sequence.

15.  $4, -4, 0, -4, -4, \dots$

16.  $100, 20, 5, 4, \frac{5}{4}, \dots$

17. Write the first five terms of the sequence  $a_1 = 3; a_n = -a_{n-1} + 5$ . Determine whether the sequence is *arithmetic*, *geometric*, *recursive*, or *none of these*. Explain your reasoning.

## 6.7 Enrichment and Extension

### Summation/Sigma Notation

*Summation notation*, or *sigma notation*, is a convenient shorthand used to write a concise expression to represent a sum with many terms. To find the sum of an infinite geometric series, first determine  $a_1$ ,  $n$ , and  $r$ , and then use the infinite series formula.

**Example:** Expand the series as a sum of terms. Then evaluate the series.

$$1. \sum_{n=1}^5 2^n = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 62$$

$$2. \sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \dots = \frac{3}{1 - \frac{1}{2}} = 6$$

**Expand the series as a sum of terms, if necessary. Then evaluate the series.**

$$1. \sum_{m=1}^6 (200 - m)$$

$$2. \sum_{k=1}^4 k(k - 1)$$

$$3. \sum_{n=1}^6 3^n$$

$$4. \sum_{n=1}^4 (3n^2 - 2)$$

$$5. \sum_{p=1}^5 (2p - 3)$$

$$6. \sum_{n=1}^9 \left(\frac{1}{2}\right)^{n-1}$$

$$7. \sum_{n=1}^7 2^{n-1}$$

$$8. \sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$$

$$9. \sum_{n=1}^{\infty} 5(0.2)^{n-1}$$

# 6.7 Puzzle Time

## What Do Cats Read For Current Events?

Write the letter of each answer in the box containing the exercise number.

**Write the first six terms of the sequence.**

1.  $a_1 = 1, a_n = a_{n-1} + 3$       2.  $a_1 = 9, a_n = a_{n-1} - 6$

3.  $a_1 = 4, a_n = 2a_{n-1}$       4.  $a_1 = -6, a_n = -\frac{1}{2}a_{n-1}$

**Write a recursive rule for the sequence.**

5. 7, 15, 23, 31, 39, ...      6. 625, 125, 25, 5, 1, ...

7. 0, -8, -16, -24, -32, ...      8. 9, -18, 36, -72, 144, ...

**Write an explicit rule for the recursive rule.**

9.  $a_1 = -2, a_n = a_{n-1} + 2$       10.  $a_1 = 14, a_n = 0.5a_{n-1}$

11.  $a_1 = -3, a_n = 6a_{n-1}$       12.  $a_1 = 5, a_n = a_{n-1} + 16$

**Write a recursive rule for the explicit rule.**

13.  $a_n = 8(4)^{n-1}$       14.  $a_n = -5n + 7$

15.  $a_n = (-10)^{n-1}$       16.  $a_n = 12n - 18$

17. The first term of a sequence is 6. Each term of the sequence is 12 less than the preceding term. Write a recursive rule for the sequence.

**Answers**

H. 4, 8, 16, 32, 64, 128

E. 1, 4, 7, 10, 13, 16

A. 9, 3, -3, -9, -15, -21

I.  $-6, 3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}$

E.  $a_1 = 0, a_n = a_{n-1} - 8$

M.  $a_1 = 2, a_n = a_{n-1} - 5$

E.  $a_1 = 625, a_n = \frac{1}{5}a_{n-1}$

D.  $a_1 = 6, a_n = a_{n-1} - 12$

S.  $a_1 = 9, a_n = -2a_{n-1}$

L.  $a_1 = -6, a_n = a_{n-1} + 12$

P.  $a_1 = 8, a_n = 4a_{n-1}$

R.  $a_1 = 1, a_n = -10a_{n-1}$

T.  $a_1 = 7, a_n = a_{n-1} + 8$

Y.  $a_n = 2n - 4$

A.  $a_n = -3(6)^{n-1}$

W.  $a_n = 16n - 11$

P.  $a_n = 14(0.5)^{n-1}$

5	3	7		17	11	4	16	9		14	1	12	8	10	2	13	6	15
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