7.8 Start Thinking

Copy and complete the table below by putting a check mark next to the type of factoring that applies to the polynomial.

Polynomial	GCF	$x^2 + bx + c$	$ax^2 + bx + c$	Difference of Squares
$8x^2 + 20x + 48$				
$x^2 - 19x + 48$				
$x^2 - 16$				
$6x^2 + 19x + 8$				

Is it possible to use more than one factoring technique to factor a polynomial? Explain.

7.8 Warm Up

Use mental math to simplify.

$$35 + 20 + 5$$
 2. $15 \bullet 7 \bullet 2$

Cumulative Review Warm Up

Evaluate the expression.

1.
$$64^{1/6}$$

2.
$$(-27)^{2/3}$$
5. $(-64)^{4/3}$

3.
$$(256)^{3/8}$$

4.
$$(\sqrt{4})^2$$

5.
$$(-64)^{4/3}$$

6.
$$216^{1/3}$$

Practice A

In Exercises 1-4, factor the polynomial by grouping.

1.
$$x^3 - 3x^2 + x - 3$$

2.
$$x^3 - 2x^2 + 9x - 18$$

3.
$$2y^3 - 2y^2 + 3y - 3$$

4.
$$3p^3 + 5p^2 - 12p - 20$$

In Exercises 5-10, factor the polynomial completely.

5.
$$4v^3 - 36v$$

6.
$$3r^2 - 8r + 7$$

7.
$$3t^3 + 12t^2 + 12t$$

8.
$$-6q^3 + 28q^2 + 10q$$

9.
$$5v^5 - 5v^4 - 10v^3$$

10.
$$7x^2 + 21x + 7$$

In Exercises 11–14, solve the equation.

11.
$$3j^3 + 21j^2 + 30j = 0$$

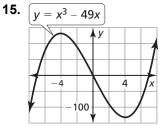
12.
$$w^4 - 36w^2 = 0$$

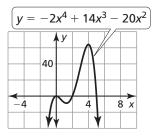
13.
$$y^3 - 2y^2 - 9y + 18 = 0$$

14.
$$5t^5 + 5t^4 - 210t^3 = 0$$

In Exercises 15 and 16, find the x-coordinates of the points where the graph crosses the x-axis.







- 17. A rectangular box has a volume of 105 cubic centimeters. The width of the rectangular box is x centimeters, the length is (2x - 3) centimeters, and the height is 3 centimeters.
 - **a.** Write a polynomial that represents the volume of the rectangular box.
 - **b.** What are the dimensions of the rectangular box?

In Exercises 18 and 19, factor the polynomial completely.

18.
$$a^3 - 4a + 3a^2b - 12b$$

19.
$$9g^3 - g - 18g^2h + 2h$$

Practice B

In Exercises 1-4, factor the polynomial by grouping.

1.
$$a^2 - 3a + ab - 3b$$

2.
$$m^2 + 7mn + 2m + 14n$$

3.
$$t^2 - 4t + tv - 4v$$

4.
$$3x^2 - 4x + 9xy - 12y$$

In Exercises 5-10, factor the polynomial completely.

5.
$$45y^4 - 20y^2$$

6.
$$8w^5 - 48w^4 + 72w^3$$

7.
$$p^3 - 3p^2 - 16p + 48$$

8.
$$12z^2 - 6z + 42$$

9.
$$-21h^4 + 77h^3 + 28h^2$$

10.
$$x^3 + 2x^2 - 49x - 98$$

In Exercises 11-14, solve the equation.

11.
$$p^3 + 2p^2 - 9p - 18 = 0$$

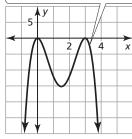
12.
$$3y^4 + 9y^3 - 120y^2 = 0$$

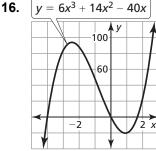
13.
$$36t - 4t^3 = 0$$

14.
$$3q^3 - 5q^2 - 27q + 45 = 0$$

In Exercises 15 and 16, find the x-coordinates of the points where the graph crosses the x-axis.

15.
$$y = -3x^4 + 18x^3 - 27x^2$$





- 17. A rectangular box has a volume of 72x cubic inches. The width of the rectangular box is x inches, the length is 3x inches, and the height is (3x - 1) inches.
 - **a.** Write a polynomial that represents the volume of the rectangular box.
 - **b.** What are the dimensions of the rectangular box?

In Exercises 18 and 19, factor the polynomial completely.

18.
$$5x^2 + 35xy - 2x - 14y$$

19.
$$5p^3 + p^2q - 15pq - 3q^2$$

7.8 Enrichment and Extension

Quadratic Form $ax^{2n} + bx^n + c$

Factoring polynomials in quadratic form uses the same methods you know from factoring quadratic polynomials, only you must now take higher powers into consideration.

Example: Factor $2x^4 - x^2 - 1$.

$$2x^{4} - x^{2} - 1 = 2x^{4} - 2x^{2} + x^{2} - 1$$

$$= (2x^{4} - 2x^{2}) + (x^{2} - 1)$$

$$= 2x^{2}(x^{2} - 1) + 1(x^{2} - 1)$$

$$= (2x^{2} + 1)(x^{2} - 1)$$

$$= (2x^{2} + 1)(x + 1)(x - 1)$$

Rewrite the *x*-term as the sum of two terms whose coefficients have product *ac* and sum *b*.

Group terms with common factors.

Factor GCF out of each pair of terms.

Distributive Property

Difference of two squares pattern

In Exercises 1–10, factor completely.

1.
$$x^6 + 11x^3 + 30$$

3.
$$3p^8 + 4p^4 - 4$$

5.
$$x^4 - 2x^2 + 1$$

7.
$$8u^6 + 10u^3 + 3$$

9.
$$-2t^8 + 7t^4 + 4$$

2.
$$y^4 - 5y^2 + 4$$

4.
$$4x^4 + 3x^2 - 1$$

6.
$$-x^{10} - 7x^5 - 10$$

8.
$$d^6 - 9d^3 + 14$$

10.
$$x^8 - 1$$



What Do You Get When You Cross A Computer With A Freezer?

Write the letter of each answer in the box containing the exercise number.

Factor the polynomial completely.

1.
$$x^3 - 3x^2 + 4x - 12$$

2.
$$6x^3 - 30x^2 + 7x - 35$$

$$3. x^2 + 2xy + 9x + 18y$$

4.
$$x^2 - 8x + xy - 8y$$

5.
$$4x^3 - 400x$$

6.
$$3x^3 + 36x^2 + 108x$$

7.
$$6x^5 + 6x^4 - 36x^3$$

8.
$$-8x^4 + 24x^3 - 88x^2$$

9.
$$x^3 - 8x^2 - 16x + 128$$

10.
$$-7x^3 - 14x^2 - 7x$$

Solve the equation.

11.
$$3x^2 - 18x + 15 = 0$$

12.
$$x^3 + 2x^2 - 36x - 72 = 0$$

13.
$$63x - 7x^3 = 0$$

14.
$$8x^3 - 3x^2 = 32x - 12$$

15. The volume of a box is 30 cubic inches. The width of the box is 2 inches less than the length. The height is 3 inches less than the length. Find the length of the box.

Answers

O.
$$4x(x+10)(x-10)$$

L.
$$(x-5)(6x^2+7)$$

R.
$$(x+4)(x-4)(x-8)$$

S.
$$(x + y)(x - 8)$$

W.
$$-7x(x+1)^2$$

E.
$$(x^2 + 4)(x - 3)$$

A.
$$6x^3(x+3)(x-2)$$

Y.
$$3x(x+6)^2$$

S.
$$-8x^2(x^2 - 3x + 11)$$

E.
$$(x + 9)(x + 2y)$$

O.
$$-2, \frac{3}{8}, 2$$

N.
$$-6, -2, 6$$