8.3 Start Thinking

Use a graphing calculator to graph the equation $f(x) = 3x^2 - x + 1$. Find the coordinates of the y-intercept. Explain how the coordinates of the y-intercept can be determined without graphing the equation.

The x-coordinate of the vertex can be found using the formula $x = \frac{-b}{2a}$ for any quadratic equation of the form $f(x) = ax^2 + bx + c$. Use the formula to find the x-coordinate of the vertex.

8.3 Warm Up

Complete the exercise.

1. Does $(4, 3)$ satisfy the equation $y = 3x^2 - x + 7$?
2. Does $(0, -1)$ satisfy the equation $y = -2x^2 + \frac{1}{2}x - 1$?
3. Does $(5, 0)$ satisfy the equation $y = 4x^2 - 2x + 4$?
4. Does $(-1, -9)$ satisfy the equation $y = -2x^2 + 3x - 4$?

8.3 Cumulative Review Warm Up

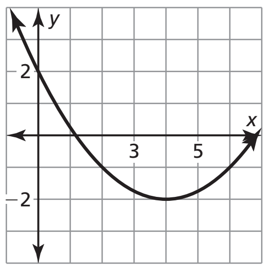
Solve the inequality. Graph the solution.

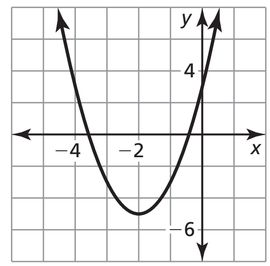
1. $4y \geq -12$
2. $36 > 6t$
3. $\frac{a}{5} > 9.3$
4. $-18 \geq \frac{9}{2}t$

Name______________________________ Date__________

8.3 Practice A

In Exercises 1 and 2, find the vertex, the axis of symmetry, and the *y*-intercept of the graph.

1. 

2. 

In Exercises 3–6, find (a) the axis of symmetry and (b) the vertex of the graph of the function.

3. $f(x) = 3x^2 - 6x$

4. $y = 5x^2 + 3x$

5. $y = -7x^2 + 14x + 1$

6. $f(x) = -4x^2 + 20x + 15$

In Exercises 7–10, graph the function. Describe the domain and range.

7. $f(x) = 3x^2 - 12x + 6$

8. $y = 5x^2 + 20x - 9$

9. $y = -6x^2 - 12x - 5$

10. $f(x) = -7x^2 + 28x - 8$

11. Describe and correct the error in finding the axis of symmetry of the graph of $y = -2x^2 + 16x + 7$.

✗ $x = -\frac{b}{2a} = -\frac{16}{2(2)} = -4$

In Exercises 12 and 13, tell whether the function has a minimum value or a maximum value. Then find the value.

12. $f(x) = 5x^2 - 20x + 3$

13. $y = -3x^2 + 12x - 7$

14. The vertex of a parabola is $(2, -2)$. Another point on the parabola is $(5, 7)$. Find another point on the parabola. Justify your answer.

In Exercises 15 and 16, use the *minimum* or *maximum* feature of a graphing calculator to approximate the vertex of the graph of the function.

15. $y = 0.2x^2 + \sqrt{6}x - 5$

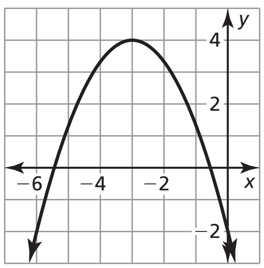
16. $y = -5.3x^2 + 3.6x + 2$

Name ____ Date __________

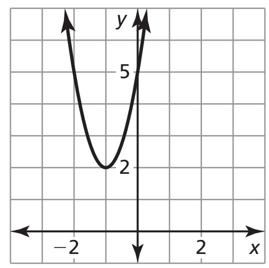
8.3 Practice B

In Exercises 1 and 2, find the vertex, the axis of symmetry, and the *y*-intercept of the graph.

1.



2.



In Exercises 3–6, find (a) the axis of symmetry and (b) the vertex of the graph of the function.

3. $f(x) = 4x^2 + 12x$

4. $y = -5x^2 - 20x + 4$

5. $y = -8x^2 + 24x + 13$

6. $f(x) = \frac{2}{3}x^2 - 6x + 15$

In Exercises 7–10, graph the function. Describe the domain and range.

7. $f(x) = 4x^2 + 8x + 11$

8. $y = -6x^2 - 12x - 7$

9. $y = \frac{1}{2}x^2 - 8x + 3$

10. $f(x) = -\frac{2}{3}x^2 + 4x + 2$

11. Describe and correct the error in finding the vertex of the graph of $y = x^2 + 6x + 2$.

> X $x = -\dfrac{b}{2a} = -\dfrac{6}{2(1)} = -3$
>
> So, the vertex is $(-3, 2)$.

In Exercises 12 and 13, tell whether the function has a minimum value or a maximum value. Then find the value.

12. $f(x) = -6x^2 + 24x - 5$

13. $y = \frac{1}{3}x^2 + 8x - 1$

In Exercises 14 and 15, use the *minimum* or *maximum* feature of a graphing calculator to approximate the vertex of the graph of the function.

14. $y = -2.1x^2 + \pi x + 3$

15. $y = 1.25x^2 - 2^{3/4}x + 3$

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8.3 Enrichment and Extension

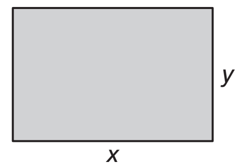
Finding Maximum Values

Maximum values are often needed in different circumstances in mathematics. How would one maximize the area of a rectangular field given 1000 feet of fencing?

Let x and y represent the length and width of the field.

Perimeter: $2x + 2y = 1000 \rightarrow y = 500 - x$

Area: $A = xy = x(500 - x) = -x^2 + 500x$



To solve a problem of maximization, you must find the y-coordinate of the vertex, which is the maximum value of the function. Use the vertex formula for the x-value and substitute the x-value into the area equation to solve for the maximum area.

$$A = -x^2 + 500x,\ x = -\frac{b}{2a} = -\frac{500}{2(-1)} = 250$$

$$A = -(250)^2 + 500(250) = 62{,}500 \text{ ft}^2$$

This is the maximum area possible.

Solve the problem.

1. Find the area of the largest possible rectangular garden that could be surrounded by 600 feet of fencing.

2. A family wants to fence in their lawn using one side of their house as one of the sides. They have 200 feet of fencing. How large is the area they can enclose?

3. A farmer is building a pen for his pigs with 180 feet of fencing. He is going to use a straight rock wall for one of the sides. What is the maximum area possible?

4. An art museum is making a rectangular garden in one corner of its courtyard. It only needs two sides of fencing, and it has 100 feet. What is the maximum area the museum can enclose?

Name ____ Date _________

8.3 Puzzle Time

What Has More Letters Than The Alphabet?

Write the letter of each answer in the box containing the exercise number.

Find the axis of symmetry and the vertex of the graph of the function.

1. $y = 2x^2 + 8x + 7$
2. $y = 6x^2 - 4x - 9$
3. $y = -\frac{3}{5}x^2 - 12x - 25$
4. $y = -\frac{9}{4}x^2 - 18x + 60$

Describe the domain and range of the function.

5. $y = 4x^2 + 16x - 7$
6. $y = -6x^2 + 48x - 40$
7. $y = -3x^2 + 12x + 9$
8. $y = 5x^2 - 40x + 60$

Tell whether the function has a minimum value or a maximum value. Then find the value.

9. $y = 3x^2 - 12x + 1$
10. $y = -4x^2 + 48x - 144$
11. $y = -\frac{1}{2}x^2 - 8x - 7$
12. $y = 2x^2 + 2x + 7$
13. The function $h(t) = -4t^2 + 20t$ represents the height (in feet) of an athlete t seconds after pole vaulting. After how many seconds does the athlete reach his or her maximum height?

Answers

H. $x = \frac{1}{3}; \left(\frac{1}{3}, -\frac{29}{3}\right)$

S. $x = -4; (-4, 96)$

I. $x = -10; (-10, 35)$

E. $x = -2; (-2, -1)$

T. 2.5

B. 5

F. maximum; 0

O. maximum; 25

E. minimum; –11

T. minimum; $\frac{13}{2}$

C. all real numbers; $y \leq 56$

P. all real numbers; $y \geq -20$

F. all real numbers; $y \geq -23$

O. all real numbers; $y \leq 21$

13	2	9		8	11	4	12		7	5	10	3	6	1