9.4 | Start Thinking

Use a graphing calculator to graph $f(x) = x^2 + 4x - 1$. Find the minimum of the function using the CALC feature on the graphing calculator. Explain the relationship between the minimum of the function and its vertex.

Graph $g(x) = -x^2 + 2x - 3$. Use the CALC feature to find the maximum of the function. Explain how to know when a quadratic function has a minimum or a maximum.

9.4 | Warm Up

Factor.

1. $x^2 - 9$

2. $y^2 + 2y + 1$

3. $4x^2 - 1$

4. $x^2 - 8x + 16$

5. $4a^2 + 4a + 1$

6. $1 - 49x^2$

7. $9a^2 + 6a + 1$

8. $4a^2 - 20a + 25$

9.4 | Cumulative Review Warm Up

Solve the equation. Check your solution.

1. $5(z - 4) = 35$

2. $3 - 6m = 27$

3. $26 = 4c + 2(7 + c)$

4. $-3 = 11y + 6(y + 8)$

5. $-3(5g + 2) = 39$

6. $-4h - 3(10 + 2h) = -10$

7. $3n - 12 = 78$

8. $n + 1 = -18$

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9.4 Practice A

In Exercises 1–3, find the value of *c* that completes the square.

1. $x^2 - 6x + c$

2. $x^2 - 10x + c$

3. $x^2 + 2x + c$

In Exercises 4–6, complete the square for the expression. Then factor the trinomial.

4. $x^2 - 4x$

5. $x^2 - 20x$

6. $x^2 + 26x$

In Exercises 7–9, solve the equation by completing the square. Round your answers to the nearest hundredth, if necessary.

7. $x^2 + 8x = 6$

8. $x^2 - 12x = -11$

9. $x^2 + 18x = 7$

10. A rectangular kitchen has an area of 160 square feet. The length is 12 feet more than the width.

    a. Write an equation that represents the area of the kitchen.

    b. Find the dimensions of the kitchen by completing the square.

In Exercises 11–16, solve the equation by completing the square. Round your answers to the nearest hundredth, if necessary.

11. $x^2 - 6x + 18 = 0$

12. $x^2 + 2x - 15 = 0$

13. $2x^2 - 16x + 20 = 0$

14. $3x^2 + 24x + 21 = 0$

15. $-4x^2 - 16x + 19 = -17$

16. $-2x^2 + 12x + 16 = 22$

17. You are completing the square to solve $5x^2 + 30x = 45$. What is the first step?

In Exercises 18–21, determine whether the quadratic function has a maximum or minimum value. Then find the value.

18. $y = x^2 - 6x - 4$

19. $y = x^2 + 8x + 10$

20. $y = -x^2 - 14x - 20$

21. $y = 2x^2 + 12x - 22$

22. The product of two consecutive even integers that are negative is 224.

    a. Write an equation to find the integers.

    b. Find the two integers.

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In Exercises 1–3, find the value of *c* that completes the square.

1. $x^2 - 16x + c$ **2.** $x^2 - x + c$ **3.** $x^2 + 7x + c$

In Exercises 4–6, complete the square for the expression. Then factor the trinomial.

4. $x^2 - 14x$ **5.** $x^2 + 30x$ **6.** $x^2 - 9x$

In Exercises 7–9, solve the equation by completing the square. Round your answers to the nearest hundredth, if necessary.

7. $x^2 + 10x = 16$ **8.** $x^2 - 3x = 7$ **9.** $x^2 + 15x = 12$

10. A wading pool is 1 foot deep and has a volume of 108 cubic feet. The width is 12 feet less than the length.

   a. Write an equation that represents the volume of the wading pool.

   b. Find the dimensions of the wading pool by completing the square.

In Exercises 11–16, solve the equation by completing the square. Round your answers to the nearest hundredth, if necessary.

11. $x^2 - 10x + 17 = 0$ **12.** $x^2 + 22x + 25 = 0$

13. $3x^2 - 15x + 27 = 0$ **14.** $2x^2 + 40x + 32 = 0$

15. $-3x^2 - 12x - 10 = -37$ **16.** $5x^2 - 15x - 10 = 20$

17. Find all values of *b* for which $x^2 + bx + 49$ is a perfect square.

In Exercises 18–21, determine whether the quadratic function has a maximum or minimum value. Then find the value.

18. $y = x^2 - 6x + 4$ **19.** $y = 2x^2 + 16x - 7$

20. $y = -3x^2 - 15x - 21$ **21.** $y = 5x^2 - 20x + 25$

22. The product of two consecutive odd integers that are positive is 323.

   a. Write an equation to find the integers.

   b. Find the two integers.

9.4 Enrichment and Extension

Vertex Form and Completing the Square

Completing the square is not just used to solve for the roots of an equation. You can also use completing the square to convert standard-form quadratics $\left(y = ax^2 + bx + c\right)$ into vertex form $\left(y = a(x - h)^2 + k\right)$.

$y = -2(x - 2)^2 + 3$

(2, 3)

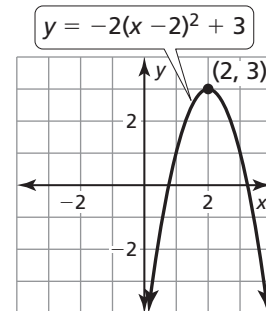**Example:** Convert $y = -2x^2 + 8x - 5$ into vertex form, state the vertex, and graph the equation.

$$y = -2x^2 + 8x - 5$$
$$= -2\left(x^2 - 4x\right) - 5 \qquad \text{Factor out } -2 \text{ from the first two terms.}$$
$$= -2\left(x^2 - 4x + (4)\right) - 5 - (-8) \qquad \text{Complete the square while subtracting value from end.}$$
$$= -2(x - 2)^2 + 3 \qquad \text{Factor and simplify equation.}$$

Convert the quadratic equation to vertex form, state the vertex, and graph the equation.

1. $y = x^2 - 2x + 6$

2. $y = x^2 + 6x + 1$

3. $y = -x^2 - 2x - 5$

4. $y = 3x^2 - 6x + 2$

5. $y = 2x^2 - 12x + 21$

6. $y = -3x^2 - 12x - 17$

7. $y = 5x^2 + 10x$

8. $y = -\dfrac{1}{2}x^2 + 4x - 7$

9. $y = \dfrac{2}{3}x^2 + 4x$

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9.4 Puzzle Time

What Does A Magician Need When He Loses His Rabbit?

Write the letter of each answer in the box containing the exercise number.

Complete the square for the expression. Then factor the trinomial.

1. $x^2 - 12x$

2. $x^2 + 18x$

3. $x^2 + 7x$

4. $x^2 - 3x$

Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

5. $x^2 + 12x = 13$

6. $x^2 - 8x = -7$

7. $x^2 + 6x = 16$

8. $x^2 - 4x - 17 = 0$

9. $3x^2 + 30x + 66 = 0$

10. $-4x^2 - 32x + 80 = 0$

Determine whether the quadratic function has a maximum or minimum value. Then find the value.

11. $y = x^2 - 6x + 4$

12. $y = -x^2 - 14x - 36$

13. A ball is thrown from a height of 5 feet with an initial velocity of 32 feet per second. The height h (in feet) after t seconds is represented by the function $h = -16t^2 + 32t + 5$. Find the maximum height of the ball.

Answers

R. $\left(x + \dfrac{7}{2}\right)^2$

A. $-8, 2$

S. $(x - 6)^2$

T. 21

R. $1, 7$

E. $-2.58, 6.58$

O. $\left(x - \dfrac{3}{2}\right)^2$

R. minimum; $(3, -5)$

E. $-13, 1$

H. maximum; $(-7, 13)$

R. $-6.73, -3.27$

A. $(x + 9)^2$

E. $-10, 2$

7		12	2	9	5		11	8	1	13	4	6	10	3