

## 2.2 Start Thinking

Consider the following statements:

*Statement 1* – “If the stove is on, then it is hot.”

*Statement 2* – “If the stove is hot, then you can cook.”

Is the statement “If the stove is on, then you can cook” true?

Write two related statements as above. Write a third statement using the beginning of your first statement and the ending of your second statement. Note if it is true or false.

## 2.2 Warm Up

**Find the common difference of the arithmetic sequence.**

**Find the next two terms.**

1. 0.09, 0.15, 0.21, ...
2. 3.36, 1.14, -1.08, ...
3. 8, 3, -2, ...
4. 2.4, 2.9, 3.4, ...
5. 2, 4, 6, ...
6. 16, 9, 2, ...

## 2.2 Cumulative Review Warm Up

**Find the area of the polygon with the given vertices.**

1.  $G(4, 1)$ ,  $H(4, -2)$ ,  $J(-1, -2)$
2.  $N(0, 0)$ ,  $P(3, 0)$ ,  $Q(3, -3)$ ,  $R(0, -1)$
3.  $K(-3, 4)$ ,  $L(0, 4)$ ,  $M(0, 0)$ ,  $N(-3, 0)$
4.  $P(-4, 4)$ ,  $Q(2, 4)$ ,  $R(2, 0)$ ,  $S(-4, 0)$

## 2.2 Practice A

**In Exercises 1 and 2, describe the pattern. Then write or draw the next two numbers or letters.**

1. 2, 5, 11, 23, 47, ...                      2. A, Z, B, Y, C, ...

**In Exercises 3 and 4, make and test a conjecture about the given quantity.**

3. the difference of any two even integers      4. the product of three negative numbers
5. An angle bisector always creates two acute angles. Find a counterexample to show that the conjecture is false.

**In Exercises 6 and 7, use the Law of Detachment to determine what you can conclude from the given information, if possible.**

6. If you go swimming, then you will get wet. You went swimming.
7. Two congruent angles have the same angle measure.  $m\angle 1 = m\angle 2$

**In Exercises 8 and 9, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible.**

8. If you study, then you will pass the exam. If you pass the exam, then you will pass the class.
9. If a straight angle is bisected, then each angle is  $90^\circ$ . If an angle is  $90^\circ$ , then it is a right angle.
10. If  $|x| = x$ , then  $x$  is positive. The value of  $x$  is 3, so  $|3| = 3$ . State the law of logic that is illustrated.

**In Exercises 11 and 12, decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.**

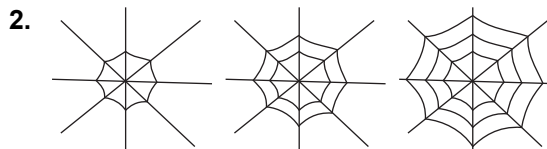
11. This weekend, the sun was shining and it did not rain. So, the next time the sun is shining, you know it will not rain.
12. The product of two even integers is always even. Because 92 and 14 are even numbers, the product is even.
13. The three tallest peaks in the Rocky Mountains are 4401 meters, 4398 meters, and 4396 meters. The three tallest peaks in the Appalachian Mountains are 2037 meters, 2026 meters, and 2025 meters. Make a conjecture that compares the Rocky Mountains to the Appalachian Mountains.
14. Use deductive reasoning to write a formula for the perimeter  $P$  of a regular polygon with  $n$  sides, where each side is  $s$ .

# 2.2

## Practice B

In Exercises 1 and 2, describe the pattern. Then write or draw the next two numbers, letters, or figures.

1. A, 26, B, 25, C, 24, ...



In Exercises 3 and 4, make and test a conjecture about the given quantity.

- 3. the sum of two absolute values
- 4. the product of a number and its square
- 5. Vertical angles are always complementary. Find a counterexample to show that the conjecture is false.

In Exercises 6 and 7, use the Law of Detachment to determine what you can conclude from the given information, if possible.

- 6. If you eat a healthy breakfast, then you will not be hungry until lunchtime. You are not hungry until lunchtime.
- 7. Adjacent angles share one common ray.  $\angle AOB$  and  $\angle DOB$  are adjacent angles.

In Exercises 8 and 9, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible.

- 8. If a polygon has three sides, then it is a triangle. If triangle has two congruent sides, then it is an isosceles triangle.
- 9. If it is Tuesday, then you mow the grass. If you mow the grass, then you water the flowers.

In Exercises 10 and 11, decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

- 10. All mammals have hair. Cats are mammals. So, all cats have hair.
- 11. Each time you go to school you walk. You went to school today, so you walked.
- 12. Is it possible to have a series of true conditional statements that lead to a false conclusion? Explain.

13. The table shows the cost per pound of several varieties of organic and nonorganic produce at your local grocery store. What conjecture can you make about the relation between the cost of organic produce and the cost of nonorganic produce? Explain your reasoning.

	Organic	Nonorganic
Bananas	\$0.49	\$0.29
Carrots	\$1.19	\$0.89
Strawberries	\$3.99	\$2.99

# 2.2

## Enrichment and Extension

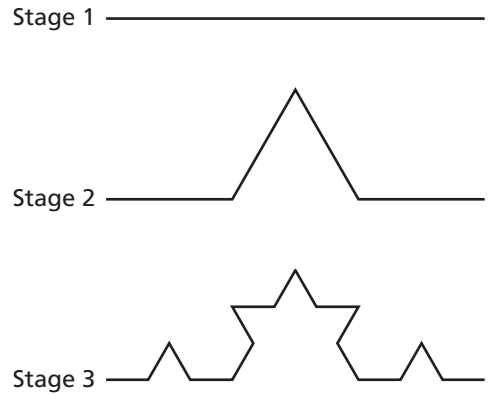
### Inductive Reasoning & Iteration

A common technique that utilizes *inductive reasoning* to find new patterns is called *iteration*. Iteration, simply put, means to repeat the process.

- Suppose someone picks a number from 1 to 100. You get to guess the number, and the person will tell you if you're too high or too low. What is the maximum number of guesses you will ever need? Using a *binary search*, you can put the possible answers into two equal (or nearly equal) parts, discard the half that does not include what you're looking for, and then repeat the process. Try picking a middle number or the average to start. So,  $\frac{100 + 1}{2} = 50.5$ , so, 50 would be a good starting number. If it is too high, use 1 and 50 and repeat the process. If it is too low, use 51 and 100 to repeat the process. Find the largest number of guesses required to find a number from 1 to 100.

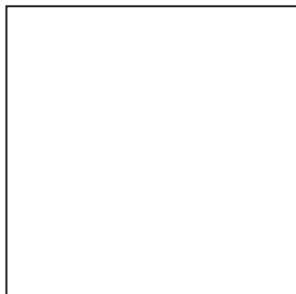
- Start with a line segment. Then, on each side, erase the middle third and add two segments the same length. Iterate this with each segment onto the new figure: erase the middle third and add two new segments of the same size. This is called a *fractal*.

Now, pretend the initial segment has a length of 1 foot. Find the total length of the shape for the first several stages. If we represent the first stage as  $n = 1$ , find an expression that models the pattern of the length at a given stage.

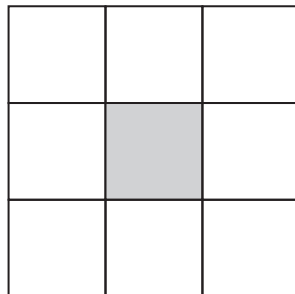


- Below is an image called *Sierpinski's Carpet*. It begins with a blank white square with side lengths of 1 foot. Then it is divided into 9 equal smaller squares with the middle square shaded. Find the fraction of the unshaded area in the first several stages. If we represent the first stage as  $n = 1$ , find an expression that models the pattern of the shaded area at a given stage.

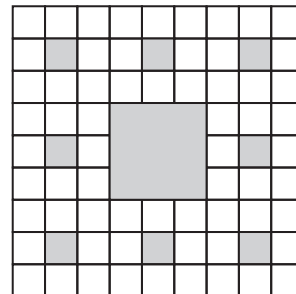
Stage 1:



Stage 2:



Stage 3:



# 2.2 Puzzle Time

## Which Garden Insects Are Always Polite?

Circle the letter of each correct answer in the boxes below. The circled letters will spell out the answer to the riddle.

**Complete the sentence.**

1. A \_\_\_\_\_ is an unproven statement that is based on observations.
2. \_\_\_\_\_ uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.
3. You use \_\_\_\_\_ when you find a pattern in specific cases and then write a conjecture for the general case.
4. A \_\_\_\_\_ is a specific case for which the conjecture is false.

**State the law of logic represented. (A) Law of Syllogism (B) Law of Detachment (C) neither**

5. If you exercise every day, then you will be a better athlete. You exercise every day. So, you will be a better athlete.
6. If you play baseball, then you play a sport. If you play a sport, then you are an athlete. You play baseball, so you are an athlete.

**Find the counterexample that makes the conjecture false.**

7.  $\frac{N}{N} = 1$
8. All prime numbers are odd.

<b>L</b> counter-example	<b>A</b> $N = 0$	<b>I</b> false term	<b>F</b> 0	<b>A</b> C	<b>N</b> proposal	<b>D</b> conjecture	<b>N</b> examples
<b>Y</b> inductive reasoning	<b>H</b> 15	<b>I</b> real world	<b>S</b> $N = -\frac{1}{2}$	<b>B</b> A	<b>U</b> deductive reasoning	<b>G</b> B	<b>S</b> 2