

4.4 Start Thinking

Find at least two objects in each of the following categories: circle, square, triangle, and rectangle (nonsquare). Use a table to compare each object of the same category in the following ways: Are all angle measures the same? Is each shape exactly the same? Are the objects the same size?

4.4 Warm Up

Plot and connect the points in a coordinate plane to make a polygon. Name the polygon.

1. $A(-3, 2)$, $B(-2, 1)$, $C(3, 3)$
2. $E(1, 2)$, $F(3, 1)$, $G(-1, -3)$, $H(-3, -2)$
3. $J(3, 3)$, $K(3, -3)$, $L(-3, -3)$, $M(-3, 3)$
4. $P(2, -2)$, $Q(4, -2)$, $R(5, -4)$, $S(2, -4)$

4.4 Cumulative Review Warm Up

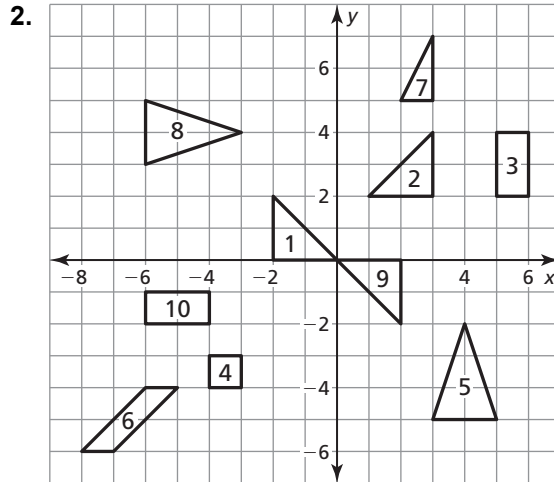
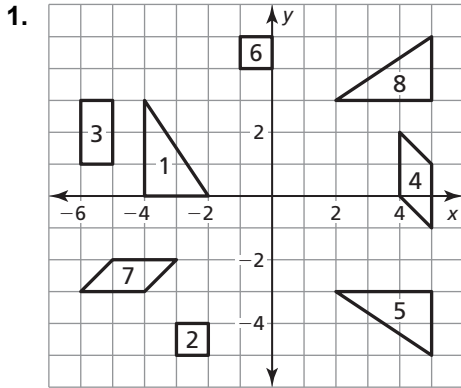
Find the coordinates of point P along the directed line segment AB so that AP to PB is the given ratio.

1. $A(4, -3)$, $B(9, -1)$; 2 to 3
2. $A(-1, -5)$, $B(7, 0)$; 4 to 1
3. $A(-1, -4)$, $B(2, 5)$; 3 to 1
4. $A(-2, 1)$, $B(6, -5)$; 3 to 5

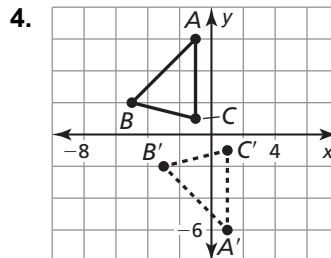
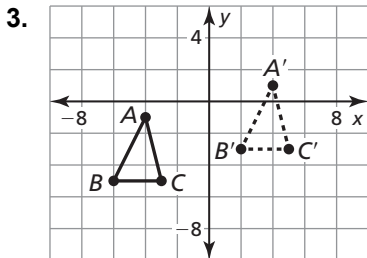
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Practice A

In Exercises 1 and 2, identify any congruent figures in the coordinate plane. Explain.

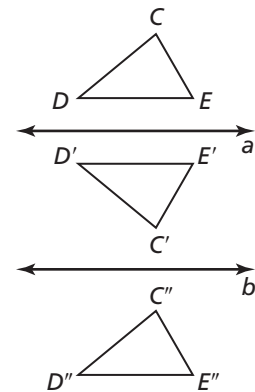


In Exercises 3 and 4, describe a congruence transformation that maps $\triangle ABC$ to $\triangle A'B'C'$.



In Exercises 5 and 6, determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.

5. $A(5, 2), B(2, 2), C(2, 7)$ and $S(-4, -5), T(-1, -5), U(-1, 0)$
6. $E(6, -2), F(10, -2), G(10, -8), H(6, -8)$ and $W(4, 8), X(4, 10), Y(8, 10), Z(8, 8)$
7. In the figure, $a \parallel b$, $\triangle CDE$ is reflected in line a , and $\triangle C'D'E'$ is reflected in line b . List three pairs of segments that are parallel to each other. Then determine whether any segments are congruent to $\overline{EE''}$.



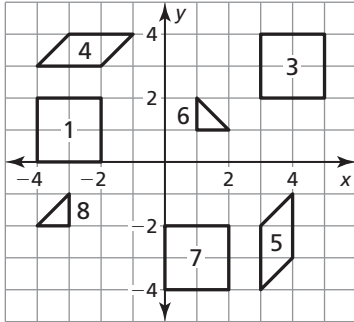
In Exercises 8 and 9, find the measure of the acute or right angle formed by intersecting lines so that P can be mapped to P'' using two reflections.

8. A rotation of 28° maps P to P'' .
9. The rotation $(x, y) \rightarrow (-y, x)$ maps P to P'' .

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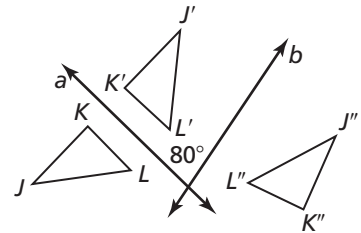
Practice B

1. Identify any congruent figures in the coordinate plane. Explain.



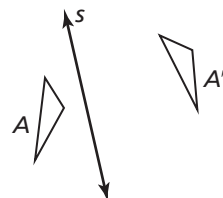
2. Determine whether the polygons with the vertices $A(0, 6)$, $B(8, 6)$, $C(6, 2)$, $D(2, 2)$ and $P(-3, -4)$, $Q(-7, -4)$, $R(-1, -8)$, $S(-5, -8)$ are congruent. Use transformations to explain your reasoning.

In Exercises 3–5, $\triangle JKL$ is reflected in line a , and $\triangle J'K'L'$ is reflected in line b .



- \overline{JK} is perpendicular to line a and has a length of 3 units, and vertex K is 1 unit from line a . Find the distance $\overline{JJ'}$.
- Find the angle of rotation that maps $\triangle JKL$ onto $\triangle J''K''L''$.
- Is \overline{JK} parallel to $\overline{J''K''}$? Explain your reasoning.
- The rotation $(x, y) \rightarrow (y, -x)$ maps P and P' . Find the measure of the acute or right angle formed by intersecting lines so that P can be mapped to P' using two reflections.
- Is it *always*, *sometimes*, or *never* true that the composition of two reflections results in the same image as a translation? Explain your reasoning.

8. $\triangle A$ is reflected in line s to form $\triangle A'$ and then reflected in line t to form $\triangle A''$. Draw line t and intermediate $\triangle A'$ to complete the figure that represents these transformations.



9. Your friend claims that if you have a series of many parallel lines, reflecting a figure in two of the lines will produce the same result as reflecting the image in four or six of the lines. Is your friend correct? Explain your reasoning.

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Enrichment and Extension

Matrix Addition and Translation

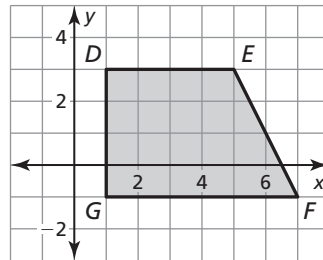
A *matrix* is a rectangular arrangement of numbers in rows and columns. (The plural of matrix is matrices.) Each number in a matrix is called an *element*. The *dimensions* of a matrix are the numbers of rows and columns. The matrix to the right has three rows and four columns, so the dimensions of the matrix are 3 x 4, read “three by four.”

$$\begin{bmatrix} 5 & 4 & 4 & 9 \\ -3 & 5 & 2 & 6 \\ 3 & -7 & 8 & 7 \end{bmatrix}$$

You can represent a figure in the coordinate plane using a matrix with two rows. The first row has the *x*-coordinates of the vertices. The second row has the corresponding *y*-coordinates. Each column represents a vertex, so the number of columns depends on the number of vertices of the figure.

Example: Write a matrix to represent point *D*.

Solution: $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ The *x*-coordinate is 1 and the *y*-coordinate is 3.



- Write a matrix to represent point *F*.
- Write a polygon matrix for *DEFG*.

To add or subtract matrices, you add or subtract corresponding elements, and the resulting matrix must have the same dimensions.

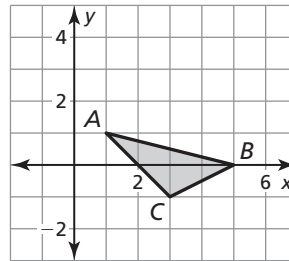
Perform the operation.

3. $\begin{bmatrix} 5 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 9 & -1 \end{bmatrix} - \begin{bmatrix} 1 & -7 & 0 \\ 4 & -2 & 3 \end{bmatrix}$

In Exercises 5–8, use the diagram.

- Write a polygon matrix for $\triangle ABC$.
- Write a matrix that, when added to the polygon matrix for $\triangle ABC$, translates the coordinates 1 unit left and 3 units up. This matrix is called a *translation matrix*.



- Add the translation matrix from Exercise 6 to the polygon matrix *ABC*. The result is called an *image matrix*, which represents the sum of a translation matrix and the matrix of a preimage.
- Graph a congruent triangle of $\triangle ABC$ translated 1 unit left and 3 units up. Label the triangle $A'B'C'$. What do you notice about the resulting coordinates?



Puzzle Time

What Geometric Figure Is Like A Lost Parrot?

Write the letter of each answer in the box containing the exercise number.

1. Complete the sentence. Two geometric figures are _____ figures if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other.
2. Congruent figures have the same size and shape. True or false?
3. Are three equilateral triangles with respective sides of 3 centimeters, 4 centimeters, and 4 inches congruent?
Yes or no?
4. A figure is reflected in line k , and the image is then reflected in line m . The measure of the acute angle formed between lines k and m is 42° . What is the angle of rotation?

Given $\triangle ABC$ with vertices $A(2, 3)$, $B(4, 3)$, and $C(4, -5)$, and the translation $(x, y) \rightarrow (x + 2, y - 1)$, find the vertex of the image.

5. A'
6. B'
7. C'
8. Complete the sentence. The polygons with vertices $A(0, 7)$, $B(0, 4)$, $C(5, 4)$, $D(5, 7)$ and $E(7, 3)$, $F(7, 0)$, $G(12, 0)$, $H(12, 3)$ _____ congruent.

Answers

- R. are not
- I. constructed
- N. $(6, -6)$
- M. yes
- A. are
- B. $(2, 6)$
- G. $(4, 2)$
- L. 84
- P. no
- E. 48
- O. true
- X. $(2, 4)$
- Y. congruent
- O. $(6, 2)$
- U. $(-6, 6)$

8		3	6	4	1	5	2	7
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