

5.8 Start Thinking

Use dynamic geometry software to create any $\triangle ABC$ in a coordinate plane such that the center of the triangle is the origin. Use the software to manipulate the triangle so it has whole-number degree angle measures.

Explain how you can use this triangle and the software to prove that knowing all three angle measures in a triangle is not enough to create a congruent triangle.

5.8 Warm Up

Find the distance between the points with the given coordinates. Round to the nearest tenth, if necessary.

- $(7, -3), (13, 7)$
- $(-1, -5), (-4, -4)$
- $(6, -11), (6, 7)$
- $(-3, 0), (4, -2)$
- $(-15, -8), (-3, -4)$
- $(10, 24), (4, 3)$

5.8 Cumulative Review Warm Up

Use the property to complete the statement.

- Substitution Property of Equality:
If $CD = 30$, then $CD + EF = \underline{\hspace{2cm}}$.
- Multiplication Property of Equality:
If $EF = GH$, then $4 \cdot EF = \underline{\hspace{2cm}}$.
- Subtraction Property of Equality:
If $PQ = AB$, then $PQ - JK = \underline{\hspace{2cm}}$.

5.8 Practice A

In Exercises 1–4, place the figure in a coordinate plane in a convenient way.

Assign coordinates to each vertex. Explain the advantages of your placement.

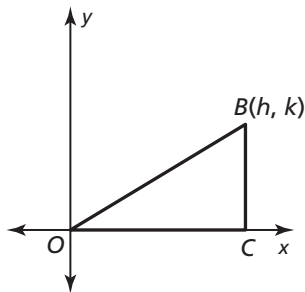
- a rectangle 2 units wide and 6 units long
- an isosceles right triangle with 4-unit legs
- a rectangle ℓ units long and w units wide
- an isosceles triangle with base length b and height h

In Exercises 5 and 6, graph the triangle with the given vertices. Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain. (Assume a and b are positive and $a \neq b$.)

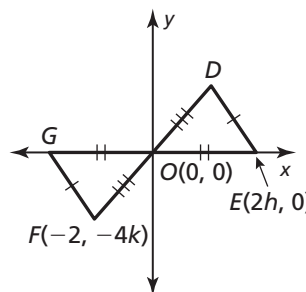
- $A(0, 0)$, $B(a, 2)$, $C(a, 0)$
- $J(0, 0)$, $K(0, a)$, $L(b, 0)$

In Exercises 7 and 8, find the coordinates of any unlabeled vertices. Then find the indicated length(s).

7. Find OB .

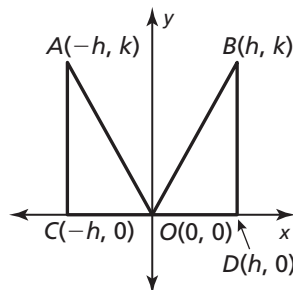


8. Find FD and DE .



9. **Given:** Coordinates of vertices of $\triangle ACO$ and $\triangle BDO$

Prove: $\triangle ACO \cong \triangle BDO$



- Your friend says that a convenient way to draw an equilateral triangle on a coordinate plane is with the base along the x -axis starting at the point $(0, 0)$. Is your friend correct? Explain your reasoning.
- You are writing a coordinate proof about right triangles with leg lengths in a ratio of 3 to 4. Assign coordinates to represent such a triangle in a coordinate plane in a convenient way, using a shorter leg length of $3a$. Then find the length of the hypotenuse.

5.8

Practice B

In Exercises 1–3, place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex. Explain the advantages of your placement.

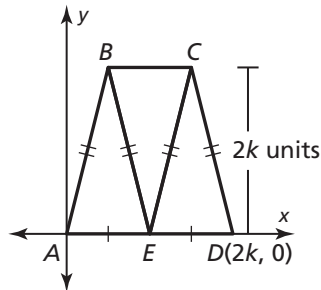
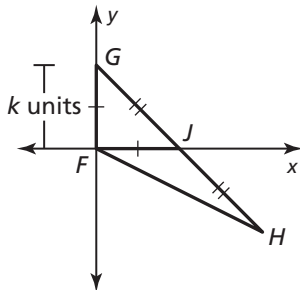
1. a rectangle twice as long as it is wide
2. a right triangle with a leg length of 3 units and a hypotenuse with a positive slope
3. an obtuse scalene triangle

In Exercises 4 and 5, graph the triangle with the given vertices. Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain.

4. $J(0, 0), K(a, b), L(2a, 0)$
5. $P(0, 0), Q(5a, 0), R(8a, 4a)$

In Exercises 6 and 7, find the coordinates of any unlabeled vertices. Then find the indicated lengths.

6. Find GH and FH .
7. Find BC and CD .



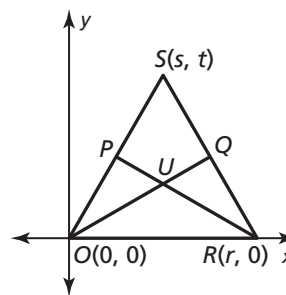
8. The vertices of a quadrilateral are given by the coordinates $W(3, 5), X(5, 0), Y(-3, -4),$ and $Z(-5, 1)$. Is the quadrilateral a parallelogram? a trapezoid? Explain your reasoning.
9. Write a coordinate proof for the following statement.

Any $\triangle ABC$ formed so that vertex C is on the perpendicular bisector of \overline{AB} is an isosceles triangle.

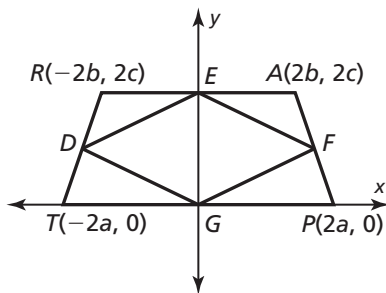
5.8 Enrichment and Extension

Coordinate Proofs

- Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Prove that angle PQR is a right angle by introducing two congruent right triangles into your diagram. Verify that the slope of \overline{QP} is the negative reciprocal of the slope of \overline{QR} .
- Prove that quadrilateral $A(1, -2)$, $B(13, 4)$, $C(6, 8)$, and $D(-2, 4)$ is a trapezoid but is *not* an isosceles trapezoid.
- In the diagram to the right, P is the midpoint of \overline{OS} and Q is the midpoint of \overline{RS} .
 - Find the coordinates of P and Q .
 - Find the equations of the line segments of \overline{PR} and \overline{QO} .

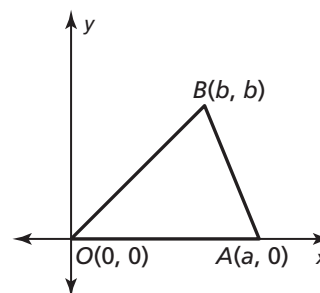


- In the diagram of trapezoid $TRAP$ below, $\overline{TR} \cong \overline{PA}$, and D , E , F , and G are midpoints of the indicated sides. Prove $DEFG$ is a rhombus.



In Exercises 5–8, give all possible coordinates for the desired point, assuming that it lies in the coordinate plane.

- If $\triangle OAB \cong \triangle OAC$ and $C \neq B$, find C .
- If $\triangle OAB \cong \triangle ODB$ and $D \neq A$, find D .
- If $\triangle OAB \cong \triangle AOF$, find F .
- If $\triangle OAB \cong \triangle BGO$, find G .





Puzzle Time

What Do Parachute Jumpers Pack Their Gear In?

Write the letter of each answer in the box containing the exercise number.

Complete the statement.

1. A(n) _____ proof involves placing geometric figures in a coordinate plane.
2. To find the length of a side of a figure, you can use the _____ formula.
3. When you use _____ to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

Place the figure in a coordinate plane and find the indicated length.

4. A right triangle has leg lengths of 20 and 21 units. Find the length of the hypotenuse.
5. An isosceles triangle has a base length of 126 units and a height of 16 units. Find the length of one of the legs.
6. A rectangle has a length of 80 units and a width of 39 units. Find the length of the diagonal.
7. A square has side length 10 units. Find the approximate length of the diagonal.

Answers

- S. 14.1
- B. 29
- Q. 15.2
- I. distance
- A. 65
- E. 74
- G. coordinate
- V. 102
- E. 28.6
- N. slope
- A. variables
- M. geometric
- C. numbers
- R. 89

5	2	6		4	3	1	7
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