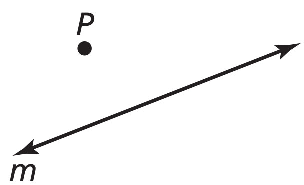
6.3 Start Thinking

Draw a right triangle and construct the three altitudes. What conclusions can you make about the three altitudes of your triangle?

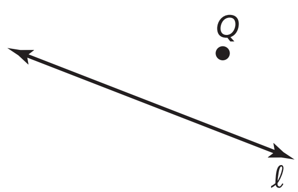
6.3 Warm Up

Use a compass and a straightedge to perform the indicated construction.

1. Construct a line perpendicular to line m through point P.



2. Construct a line perpendicular to line ℓ through point Q.



6.3 Cumulative Review Warm Up

Write an equation of the line passing through point *P* that is perpendicular to the given line.

1. $P(-2, 4), y = -\frac{2}{3}x + \frac{5}{2}$

2. $P(5, 11), y = 8$

3. $P\left(\frac{3}{4}, -9\right), y = x$

4. $P(1, -7), y = 2x + 3$

5. $P(3, -2), 3x - 5y = 4$

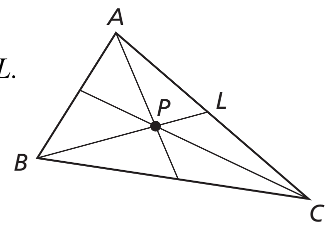
6. $P\left(-\frac{1}{2}, -\frac{3}{2}\right), x = -3$

Name______________________________ Date__________

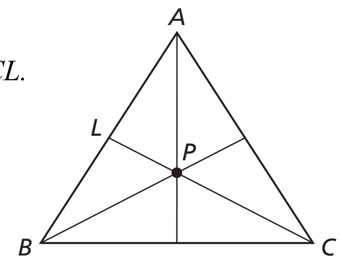
6.3 Practice A

In Exercises 1–4, point P is the centroid of $\triangle ABC$. Use the given information to find the indicated measures.

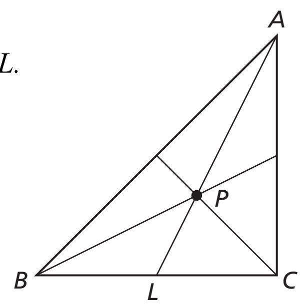
1. $BL = 12$
Find BP and PL.



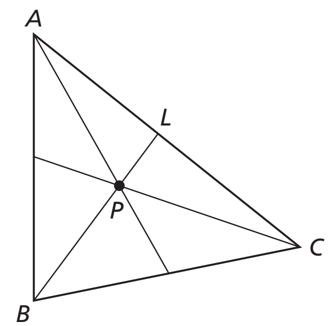
2. $CP = 16$
Find PL and CL.



3. $AL = 27$
Find AP and PL.



4. $BP = 102$
Find PL and BL.



In Exercises 5 and 6, find the coordinates of the centroid of the triangle with the given vertices.

5. $Q(-2, 6), R(4, 0), S(10, 6)$

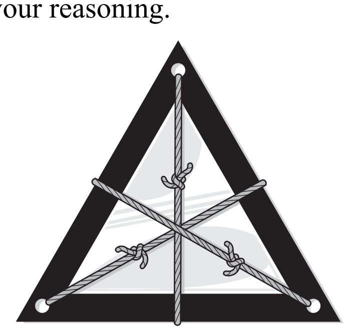
6. $U(3, 3), V(5, -1), W(-2, 1)$

In Exercises 7 and 8, tell whether the orthocenter is *inside*, *on*, or *outside* the triangle. Then find the coordinates of the orthocenter.

7. $J(1, 3), K(-3, 1), L(0, 0)$

8. $D(-3, -2), E(-2, -2), F(1, 2)$

9. To transport a triangular table, you remove the legs. You secure the glass top to the frame by looping a string from a hole in each vertex around the opposite side, then pulling it tight and tying it. At what point of concurrency do the three strings intersect? Explain your reasoning.



10. Your friend claims that it is impossible for the centroid and the orthocenter of a triangle to be the same point. Is your friend correct? Explain your reasoning.

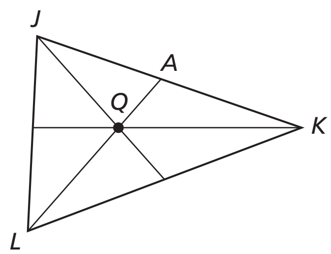
Name ____ Date __________

6.3 Practice B

In Exercises 1–3, point *Q* is the centroid of $\triangle JKL$. Use the given information to find the indicated segment lengths.

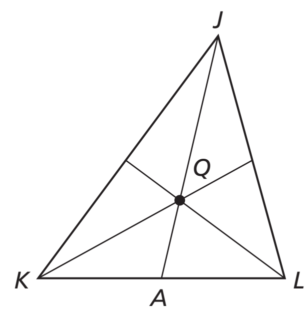
1. $AQ = 21$

   Find QL and AL.



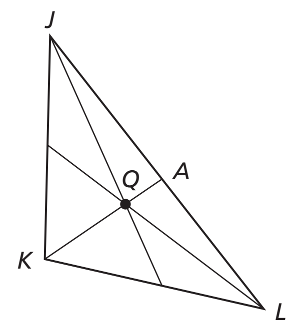
2. $JA = 72$

   Find JQ and QA.



3. $KQ = 10$

   Find QA and KA.



4. Find the coordinates of the centroid of the triangle with the vertices $A(-6, 8)$, $B(-3, 1)$, and $C(0, 3)$.

In Exercises 5 and 6, tell whether the orthocenter is *inside*, *on*, or *outside* the triangle. Then find the coordinates of the orthocenter.

5. $Q(-1, 5)$, $R(4, 3)$, $S(-1, -2)$

6. $L(4, 6)$, $M(-3, 2)$, $N(-2, -6)$

7. Given two vertices and the centroid of a triangle, how many possible locations are there for the third vertex? Explain your reasoning.

8. Given two vertices and the orthocenter of a triangle, how many possible locations are there for the third vertex? Explain your reasoning.

9. The centroid of a triangle is at $(2, -1)$ and vertices at $(3, -5)$ and $(-7, -4)$. Find the third vertex of the triangle.

10. The orthocenter of a triangle is at the origin, and two of the vertices of the triangle are at $(-5, 0)$ and $(3, 4)$. Find the third vertex of the triangle.

11. Your friend claims that it is possible to draw an equilateral triangle for which the circumcenter, incenter, centroid, and orthocenter are not all the same point. Do you agree? Explain your reasoning.

12. Your friend claims that when the median from one vertex of a triangle is the same as the altitude from the same vertex, the median divides the triangle into two congruent triangles. Do you agree? Explain your reasoning.

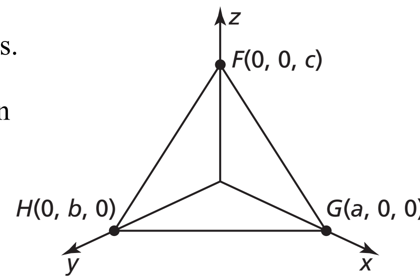
13. Can the circumcenter and the incenter of an obtuse triangle be the same point? Explain.

Name______________________________ Date__________

6.3 Enrichment and Extension

Medians in Triangles

The location of the centroid N for a triangle in three-dimensional space is calculated by averaging the x-, y-, and z-coordinates of the three points.

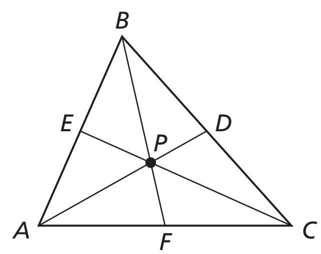


1. If $R(x_1, y_1, z_1)$, $S(x_2, y_2, z_2)$, and $T(x_3, y_3, z_3)$, find the location of the centroid N in $\triangle RST$.

2. Find the centroid N for the triangle in the figure, with vertices on the x-, y-, and z-axes.

3. The midpoint of $\overline{GH}$ is T. Calculate the coordinates of T. Then prove that $FN = 2 \bullet NT$.

In Exercises 4–6, point *P* is the centroid of $\triangle ABC$. Use the given information to find the value(s) of *x*.



4. $AP = x^2 - 2, PD = 2x - 3$

5. $CP = x^2 + 1, PE = 4x - 3$

6. $CP = 3x + 5, CE = x^2 + 2$

In Exercises 7 and 8, use the following information to find the area of the triangle described.

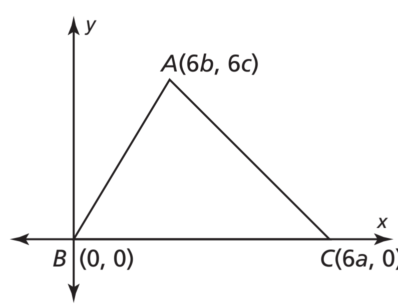
The formula below can be used to find the area A of a triangle using the measures of the medians m.

$$A = \frac{4}{3}\sqrt{s(s - m_1)(s - m_2)(s - m_3)}, \text{ where } s = \frac{1}{2}(m_1 + m_2 + m_3).$$

7. $m_1 = 19, m_2 = 17, m_3 = 10$

8. $m_1 = 1.2, m_2 = 3.4, m_3 = 4.2$

9. Given the diagram to the right, find the equations of the medians of the triangle. Use M as the median from vertex A to $\overline{BC}$, N as the median from vertex B to $\overline{AC}$, and P as the median from vertex C to $\overline{AB}$. Then find their intersection point.



Name ____ Date __________

6.3 Puzzle Time

What Did The Librarian Use For Bait When She Went Fishing?

Circle the letter of each correct answer in the boxes below. The circled letters will spell out the answer to the riddle.

Complete the sentence.

1. A(n) ________ of a triangle is a segment from a vertex to the midpoint of the opposite side.

2. The lines containing the medians of a triangle are concurrent. The point of concurrency, called the ________, is inside the triangle.

3. The centroid of a triangle is two-thirds of the distance from each vertex to the ___________ of the opposite side.

4. A(n) _________ of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

5. The lines containing the altitudes of a triangle are concurrent. This point of concurrency is the ___________ of the triangle.

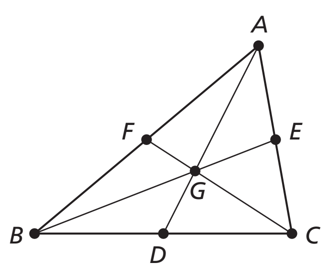
Find the indicated measurement using the diagram as a reference. Point *G* is the centroid.

6. $FC = 36$; Find GC.

7. $FC = 36$; Find FG.

8. $GF = 9$; Find FC.

9. $GF = 9$; Find GC.



Z 9	**A** 27	**R** 15	**U** point	**L** congruent	**B** 18	**I** straight	**M** 3	**O** 12	**O** orthocenter
K centroid	**G** 21	**E** center	**F** 1	**W** 24	**O** median	**I** bisector	**S** 0	**R** altitude	**M** midpoint