7.4 Start Thinking

A rhombus and a square are both quadrilaterals with four congruent sides, but a square always contains four right angles. Examine the diagrams below and determine some other distinctive characteristics of the rhombus and the square.



7.4 Warm Up

Use the diagrams to determine the measure of each angle.



7.4 Cumulative Review Warm Up

Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

- **1.** An isosceles triangle is a right triangle.
- **2.** A right triangle is a scalene triangle.
- **3.** An equilateral triangle is an equiangular triangle.
- **4.** A right triangle is an equilateral triangle.

7.4 Practice A

In Exercises 1–5, the diagonals of rhombus ABCD intersect at E. Given that $m \angle EAD = 67^{\circ}$, CE = 5, and DE = 12, find the indicated measure.

1. $m \angle AED$ В С **2.** $m \angle ADE$ 5 Ε **3.** $m \angle BAE$ 12 ′67° **4.** *AE* Ā D **5.** *BE*

In Exercises 6 and 7, find the lengths of the diagonals of rectangle JKLM.

7. JL = 2x - 66. JL = 3x + 4 $KM = \frac{3}{2}x + 1$ KM = 4x - 1

In Exercises 8 and 9, decide whether quadrilateral WXYZ is a rectangle, a rhombus, or a square. Give all names that apply. Explain your reasoning.

- **8.** W(3, 1), X(3, -2), Y(-5, -2), Z(-5, 1)**9.** W(4, 1), X(1, 4), Y(-2, 1), Z(1, -2)
- **10.** Use the figure to write a two-column proof.
 - Given: *PSUR* is a rectangle.

$$\overline{PQ} \cong \overline{TU}$$

Prove: $\overline{OS} \cong \overline{RT}$



- **11.** In the figure, all sides are congruent and all angles are right angles.
 - **a.** Determine whether the quadrilateral is a rectangle. Explain your reasoning.
 - **b.** Determine whether the quadrilateral is a rhombus. Explain your reasoning.
 - **c.** Determine whether the quadrilateral is a square. Explain your reasoning.
 - **d.** Find $m \angle AEB$.
 - **e.** Find $m \angle EAD$.



7.4

Practice B

In Exercises 1 and 2, decide whether quadrilateral *JKLM* is a rectangle, a rhombus, or a square. Give all names that apply. Explain your reasoning.

1. J(3, 5), K(7, 6), L(6, 2), M(2, 1)**2.** J(-4, -1), K(-1, 5), L(5, 2), M(2, -4)

In Exercises 3–7, the diagonals of rhombus *ABCD* intersect at *M*. Given that $m \angle MAB = 53^{\circ}$, MB = 16, and AM = 12, find the indicated measure.

- **3.** *m∠AMD*
- **4.** $m \angle ADM$
- **5.** *m∠ACD*
- **6.** *DM*
- **7.** *AC*
- 8. Find the point of intersection of the diagonals of the rhombus with vertices (-1, 2), (3, 4), (5, 8), and (1, 6).
- 9. Use the figure to write a two-column proof.
 Given: WXYZ is a parallelogram.
 ∠XWY ≅ ∠XYW

Prove: *WXYZ* is a rhombus.

- **10.** Your friend claims that you can transform every rhombus into a square using a similarity transformation. Is your friend correct? Explain your reasoning.
- **11.** A quadrilateral has four congruent angles. Is the quadrilateral a parallelogram? Explain your reasoning.
- **12.** A quadrilateral has two consecutive right angles. If the quadrilateral is not a rectangle, can it still be a parallelogram? Explain your reasoning.
- **13.** Will a diagonal of a rectangle ever divide the rectangle into two isosceles triangles? Explain your reasoning.



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7.4 Enrichment and Extension

Properties of Special Parallelograms

In Exercises 1–3, determine whether the quadrilateral can be a parallelogram. If not, write *impossible*. Explain.

- 1. The diagonals are congruent, but the quadrilateral has no right angles.
- 2. Each diagonal is 3 centimeters long and the two opposite sides are 2 centimeters long.
- 3. Two opposite angles are right angles, but the quadrilateral is not a rectangle.

In Exercises 4–7, use the information given in the diagram to solve for the missing variable.

- **4.** Find the value of *w*.
- **5.** Find the value of *x*.
- 6. Find the value of *y*.
- **7.** Find the value of *z*.



8. In *LMNP* shown at the right, $m \angle MLN = 32^\circ, m \angle NLP = (x^2)^\circ,$ $m \angle MNP = 12x^\circ, \text{ and } \angle MNP \text{ is}$ an acute angle. Find $m \angle NLP$.



9. Write a coordinate proof of part of Theorem 7.13 (*Hint:* write the vertices in terms of *a* and *b*.)

Given: DFGH is a parallelogram. $\overline{DG} \cong \overline{HF}$

Prove: *DFGH* is a rectangle.





What Do You Have To Know To Get Top Grades In Geometry?

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Write the letter of each answer in the box containing the exercise number.

Complete the sentence.

- 1. A rhombus is a parallelogram with _____ congruent sides.
- **2.** A rectangle is a parallelogram with four _____ angles.
- **3.** A square is a _____ with four congruent sides and four right angles.
- **4.** A parallelogram is a rhombus if and only if its ______ are perpendicular.
- **5.** A parallelogram is a rhombus if and only if each diagonal ______ a pair of opposite angles.
- 6. A parallelogram is a rectangle if and only if its diagonals are ______.

Decide whether each is a *rhombus*, *rectangle*, *square*, *none* of these, or *all* of these.

- **7.** A(-8, -3), B(-5, 3), C(1, 0), D(-2, -6)
- **8.** A(-6, -3), B(-6, -8), C(-2, -5), D(-2, 0)
- **9.** A(-7, 1), B(-4, -4), C(2, 2), D(-3, 4)

Given	rhombus	ABCD. fin	d the mea	asure of the	e indicated	angle in degre	es.
0		, 			maioatoa	angle in acgr.	/00.

10. $m \angle A = 119^\circ$. Find $m \angle B$.

Find the length of the diagonals of rectangle *QRST* given the following information.

- **11.** QS = 4x + 6, RT = 6x 4
- **12.** QS = 9x + 12, RT = 11x 10

7	2	5	9	4	11	1	12	8	3	10	6

E.61H.diagonalsG.twoA.fourA.squareT.kiteL.parallelogramT.noneN.111O.acuteL.rightD.intersects	Answers									
G. twoA. fourA. squareT. kiteL. parallelogramT. noneN. 111O. acuteL. rightD. intersects										
 A. square T. kite L. parallelogram N. 111 D. acute D. intersects 										
L. parallelogramT. noneN. 111O. acuteL. rightD. intersects										
N. 111 O. acute L. right D. intersects										
L. right D. intersects										
G. rhombus R. rectangle										
L. bisects M. all										
E. 26 S. congruent										
O. angles P. 119										
I. 5 U. 11										
R. perpendicular										