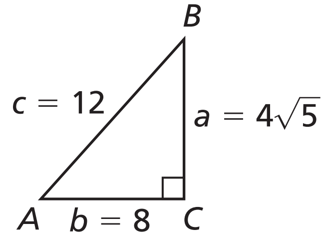
9.7 Start Thinking

Use the triangle in the diagram to find the value of each ratio. How do the three ratios relate? Do you think this relationship would be the same if the triangle was not a right triangle?



1. $\dfrac{a}{\sin A}$

2. $\dfrac{b}{\sin B}$

3. $\dfrac{c}{\sin C}$

9.7 Warm Up

Solve the proportion. Round your answer to the nearest tenth.

1. $\dfrac{a}{\sin 28^\circ} = \dfrac{21}{\sin 65^\circ}$

2. $\dfrac{15}{\sin 40^\circ} = \dfrac{c}{\sin 94^\circ}$

3. $\dfrac{b}{\sin 9^\circ} = \dfrac{63}{\sin 105^\circ}$

4. $\dfrac{54}{\sin B} = \dfrac{61}{\sin 73^\circ}$

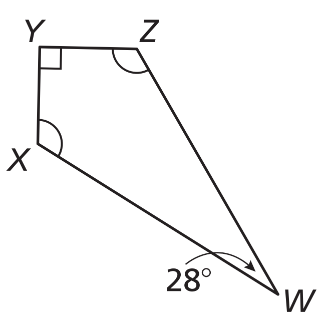
5. $\dfrac{16}{\sin 81^\circ} = \dfrac{15}{\sin A}$

6. $\dfrac{110}{\sin C} = \dfrac{85}{\sin 36^\circ}$

9.7 Cumulative Review Warm Up

Find the measures of $\angle X$ and $\angle Z$.

1.



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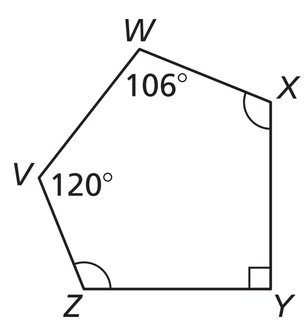

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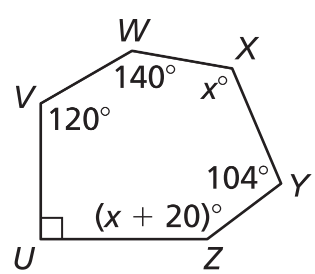


Name____ Date__________

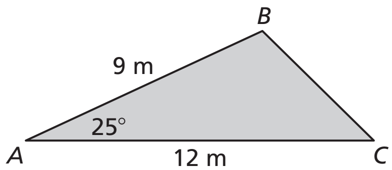
9.7 Practice A

In Exercises 1–3, use a calculator to find the trigonometric ratio. Round your answer to four decimal places.

1. cos 115° **2.** tan 95° **3.** sin 148°

In Exercises 4 and 5, find the area of the triangle. Round your answer to the nearest tenth.

4.



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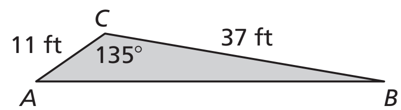
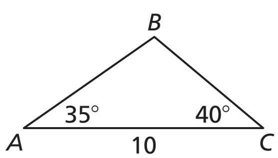


6. Place each triangle case into one of the three categories according to the first step in solving the triangle.

Law of Sines	Law of Cosines	Neither
AAA AAS	ASA SSS	SSA SAS

In Exercises 7–12, solve the triangle. Round decimal answers to the nearest tenth.

7.



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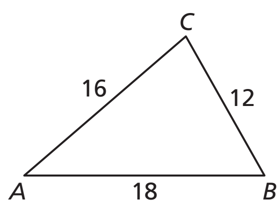

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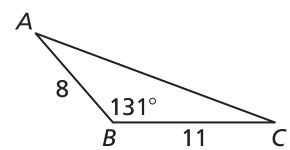

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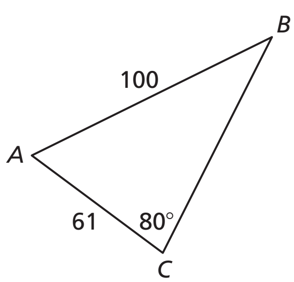

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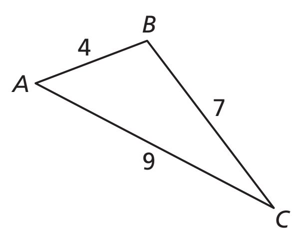

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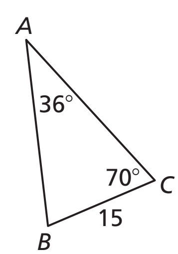
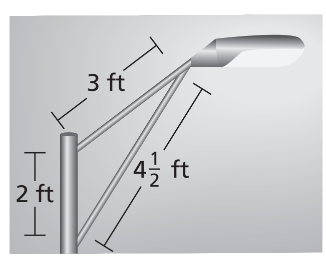


13. Determine the measure of angle *A* in the design of the streetlamp shown in the diagram.



Name ____ Date __________

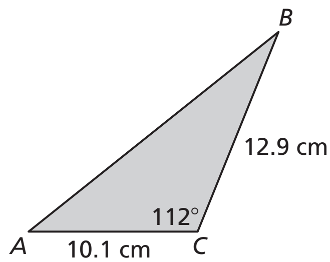
9.7 Practice B

In Exercises 1–3, use a calculator to find the trigonometric ratio. Round your answer to four decimal places.

1. $\tan 133^\circ$ **2.** $\cos 128^\circ$ **3.** $\sin 91^\circ$

In Exercises 4 and 5, find the area of the triangle. Round your answer to the nearest tenth.

4.



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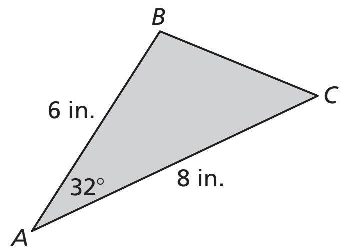
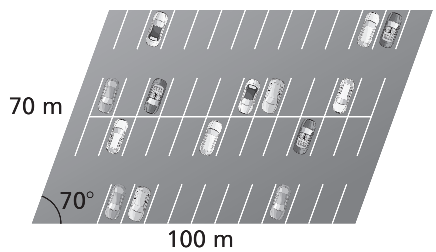
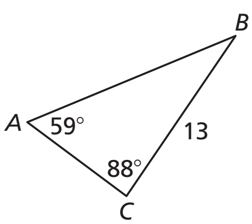


6. A parking lot has the shape of a parallelogram, as shown. Explain how you can find the area of the parking lot without using right triangles. Then find the area of the parking lot.



In Exercises 7–12, solve the triangle. Round decimal answers to the nearest tenth.

7.



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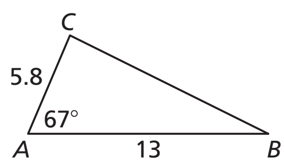

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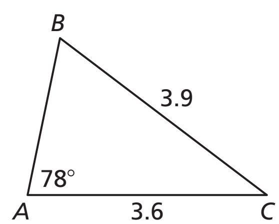

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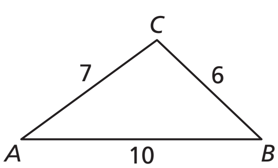

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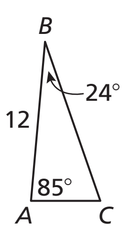

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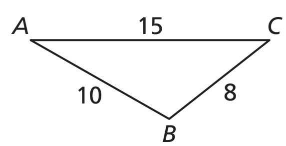
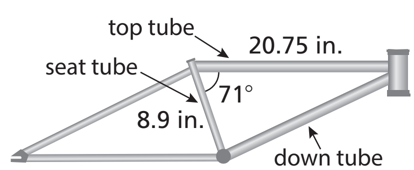


13. A bike frame has a top tube length of 20.75 inches, a seat tube length of 8.9 inches, and a seat tube angle of 71°.

a. Find the approximate length of the down tube.

b. Find the angle between the seat tube and down tube.



Name____ Date__________

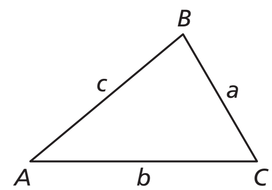
9.7 Enrichment and Extension

Law of Tangents

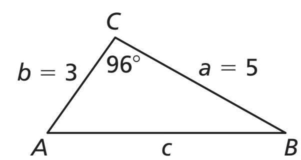
In addition to the Law of Sines and the Law of Cosines, there is another formula, the *Law of Tangents*, that can be used to solve the missing sides and angles of a triangle.

The Law of Tangents can be written as

$$\frac{a+b}{a-b} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}} \text{ or } \frac{a-b}{a+b} = \frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}},$$



where a and b are arbitrary sides and A and B are arbitrary angles.



Example: Solve the triangle to the right using the Law of Tangents.

Solution: Because $A + B + C = 180°$,

$$A + B = 180° - C = 180° - 96° = 84°,$$

$$\text{so, } \frac{A+B}{2} = 42°.$$

$\frac{a-b}{a+b} = \frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}} = \frac{5-3}{5+3} = \frac{\tan\frac{A-B}{2}}{\tan 42°}$	Substitution
$\frac{2}{8}\tan 42° = \tan\frac{A-B}{2}$	Multiply each side by tan 42°.
$0.2251 = \tan\frac{A-B}{2}$	Simplify.
$12.7° = \frac{A-B}{2}$	Take the inverse of each side.
$25.4° = A - B$	Multiply each side by 2.

Using the equations $A - B = 25.4°$ and $A + B = 84°$, and solving the systems of equations, you have $A = 54.7°$ and $B = 29.3°$. You can find the remaining side c by using the Law of Sines: $\frac{c}{\sin C} = \frac{a}{\sin A} \rightarrow c = \frac{a \sin C}{\sin A} = \frac{5 \sin 96°}{\sin 54.7°} \approx 6.09.$

In Exercise 1–3, use the Law of Tangents to solve the triangle. Round your answer to the nearest tenth.

1. In $\triangle ABC$, a is 52, b is 28, and $m\angle C = 80°$.

2. In $\triangle ABC$, c is 14, b is 9, and $m\angle A = 62°$.

3. In $\triangle ABC$, c is 20, b is 13, and $m\angle A = 66°$.

Name ______________________________ Date __________

9.7 Puzzle Time

Why Should You Always Walk A Mile In People's Shoes Before You Criticize Them?

A	B	C	D	E	F
G	H	I	J	K	L

Complete each exercise. Find the answer in the answer column. Write the word under the answer in the box containing the exercise letter.

0.8391	FEET
−0.8391	A
false	SOCKS
included	YOU'LL
−0.1763	TO
cotangent	AND
16.8	SHOES
8.9	ROAD
true	BECAUSE
0.9063	AND
129.6	END
173.7	HAVE

Complete the sentence.

A. If $\triangle ABC$ has sides of length a, b, and c, then the following are true:
$a^2 = b^2 + c^2 - 2bc\cos A,\ b^2 = a^2 + c^2 - 2ac\cos B,$ and
$c^2 = a^2 + b^2 - 2ab\cos C.$ True or false?

B. The Law of ______ can be used to solve triangles when two sides and the included angle are known, or when all three sides are known.

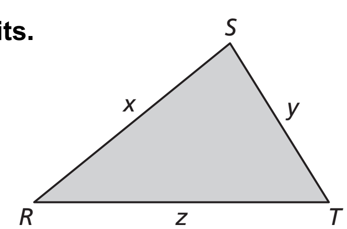
C. The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their ______ angle.

D. The Law of ______ can be used to solve triangles when two angles and the length of any side are known, or when the lengths of two sides and an angle opposite one of the two sides are known.

Find the trigonometric ratio. Round your answer to four decimal places.

E. tan 140° **F.** sin 170° **G.** cos 135° **H.** sin 115°

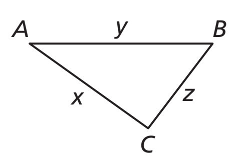
Find the area of the triangle in square units. Round your answer to the nearest tenth.



I. $x = 21, z = 18, m\angle R = 44°$

J. $x = 26, y = 15, m\angle S = 63°$

Solve for the indicated measure. Round decimal answers to the nearest tenth.



K. $x = 11, y = 14, m\angle A = 40°$; Find z.

L. $x = 15, y = 24, m\angle C = 98°$; Find z.

−0.4226	THE
14.9	SO
sines	BE
0.1736	MILE
opposite	PATH
9.0	THEIR
cosines	THEN
−0.7071	AWAY
tangent	HURT
131.3	YOU'LL
174.0	NICE